DESIGN OF ROBUST REDUCED ORDER DIGITAL AUTOPILOT FOR LAUNCH VEHICLE

by

VEENA SINHA

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

DECEMBER, 1996

DESIGN OF ROBUST REDUCED ORDER DIGITAL AUTOPILOT FOR LAUNCH VEHICLE

A Thesis Submitted

in Partial Fulfillment of the Requirements

for the Degree of

Master of Technology

by

Veena Sinha

to the

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

December 1996

Certificate

It is certified that the work contained in the thesis entitled **DESIGN OF ROBUST REDUCED ORDER DIGITAL AUTOPILOT FOR LAUNCH VEHICLE**, by Veena Sinha, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

December 1996

K. E. Holé
Professor
Department of Electrical Engineering
I.I.T. Kanpur

5/12/96

Abstract

In this thesis, a robust optimal controller for the launch vehicle is designed. Also, performance of a hybrid closed loop system containing continuous time plant and discrete time controller with and without lifting is considered. Two cases are considered:

case I : Controller designed for discrete-time model of the plant.

case II : Controller obtained by descretizing the continuous time controller.

The controllers are designed using the reduced order model of the launch vehicle. Controller order is further reduced by balanced realization truncation and frequency weighted balanced realization truncation techniques. A comparison is made of the performance of the system for different controllers with and without lifting the system.Also, a comparison is made of the performance of the various reduced order controllers with the previously designed controller for the same system. The comparison clearly indicates the benefits of lifting the system. It is found that performance with the 2^{nd} order controller in frequency weighted balanced realization for case II is better than any other controller in two different realization presented for different order. Bounds on allowed perturbations in system matrices are determined. Bounds are calculated with and without lifting the system. Larger perturbation bounds are obtained with lifting than without lifting.

Dedicated To

My Family Members

Acknowledgements

I take this opportunity to express my sincere gratitude towards my thesis supervisor, Dr. K. E. Holé for his constant guidance and encouragement throughout the preparation of this thesis. Without his help, this work would never have been completed.

I would like to thank all my friends for their good company. I am thankful to my husband, Rajeev, without whose loving attention and support, I would not have achieved this success.

Lastly, words do not suffice to express my indebtedness and the deep affection I have for my parents and all my family members, for they have been my source of inspiration and moral support.

Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

1.1 General

Due to rapid development in the technology of digital computers and microprocessors, considerable attention has been focused on the study of digital and sampled-data control systems. For such systems to obtain the best possible performance, it is often necessary to sample the signals for the various sensors and control effectors at different rates. A sampled-data system is considered a multirate system if sampling at different locations occurs at different rate.

The two primary operations in multirate processing are decimation and interpolation. They enable the data rate to be altered without significant, undesirable effects of errors such as quantization and aliasing. Decimation reduces the sampling rate effectively. Interpolation on the other hand increses the sampling rate. Fast sampling of the system results in periodic, discrete-time, linear and time-varying system. Fast sampling of the system at a multiple of the sampling frequency followed by lifting allows capturing of the systems intersample behaviour and yields a time invariant single rate system. This then permits standard order reduction ideas to be applied. Lifting technique is given in [1].

However, in [2], [3] general multirate sampled-data system is considered. Here all the inputs and outputs sampled at different rates are considered. It is shown that multiinput, multioutput, multirate sampled-data systems, while not generally periodic, are shift varying and, for any given multirate system, obtains an explicit expression for

a state space realization of its shift invariant equivalent.

Multirate sampling schemes becomes necessary for systems with special data transmission links or special sensors and actuators. They are useful for improving the response of systems in which measurements are obtained at slow rate e.g. when lab instruments are used. Multirate systems allows better control of what happens between sampling instants. In multivariable systems it may also be advantageous to have different sampling rates in different loops to reduce the computational load and to improve the numeric conditioning. Use of multirate sampling is also natural in multiprocessor systems [4]. High quality data acquisition and storage system takes advantage of multirate technique to avoid the use of expensive anti-aliasing analogue filters and to handle efficiently signals of different bandwidths which require different sampling frequencies [5].

Simple linear controllers are normally preferred over complex linear controllers. For this reason, there is a desire to have methods available for designing low order controllers for high order plants. Such methods can be divided into two classes, direct, in which the parameters defining a low order controllers are computed by some optimization or other procedure; and indirect, in which a high order controller is first found, and then a procedure used to simplify it [6].

Any mathematical model, no matter how accurate, can only approximate the true behaviour of the system. This is because of various reasons such as linearization of nonlinear system about an operating point, order reduction and neglected coupling terms. Further, the parameters involved in such a description are often subjected to variations and uncertainties for several reasons such as environmental condition, ageing, wearing and manufacturing tolerance. It is therefore important to assess the robustness of the system stability to parameter variation.

1.2 Review Of The Existing Work

Although study of systems containing multiple samplers or hold operating at different rates began in the 1950's, it has received renewed attention. Interest is partly due to

the importance of such control systems in practice since a very large number of "real-world" digital control systems are multirate. Sklansky and Ragazzini [7], developed the frequency decomposition method for determining the transfer function of multirate system. The switch decomposition method, developed by Kranc [8], is an extension of the frequency decomposition method which allows rational sampling rate ratios. Ragazzini and Franklin [9], includes an excellent review of the frequency decomposition and switch decomposition methods.

Kalman and Bertram [10], presented a method for forming a discrete time state model of an multirate (MR) sampled data plant. A frequency domain approach for discrete-time periodic time-varying systems appears in the work of Davis [11], Jury and Mullin [12], and Meyer and Burrus [13]. Nagy and Foias [14], have shown 'categorical' equivalence between periodic discrete-time systems and certain kinds of linear time invariant (LTI) systems.

Based on the results of [14] and [15], Khargonekar et al. [1], derived transfer function for periodic, discrete-time, linear, time-varying systems. It is shown that lifting has the effect of removing blocking zeros. They, then, applied this observation to several robust control problems, proving in particular that, for a single input single output (SISO) bicausal plant, a periodic controller can be designed so that the resulting system has an arbitrarily large gain margin. In [16], [17] it is shown that there is a limit to the gain margin achievable by a time invariant controller if the plant has an unstable pole and zero.

To analyze the behaviour of continuous time periodic system Bamieh et al. [18], used a lifting technique similar to that used for discrete time periodic systems in [1]. They have shown there that lifting technique is applicable to all norm based optimization problems, and in particular to sampled-data versions of the quadratic regulator and optimal filtering problem. Yamamoto [19], also used lifting for sampled-data systems, but he lifted the state as well as the input and output.

Many researchers have considered the control system design problem for multi rate sampled data plant (MRSDP) under various assumptions on the sampling rates and

the type of control used. In [20] authors analyze multirate pole placement assuming that the inputs and outputs can be paired off into loops having the same rates. Successive loop closure [21] is a heuristic technique applicable under the same assumption. An Linear Quadratic Gaussian (LQG) design method for pairwise single rate plant is reported in [22] and uses a mechanism from [10].

In [23], Franklin and Rahmani introduced a new optimal multirate control of linear continuous-time periodic systems. The control law is based on the continuous time linear quadratic regulator (LQR) problem. With the input being constrained to a certain piecewise constant signal. It is shown that multirate control scheme, when applied to LTI systems, can produce better response characteristics and a less LQR cost than that of single rate sampling if the state is sampled at the same rate in both the schemes.

Another simplifying assumption is that all the inputs (respectively, output) are updated at a single rate. Another is so called "dual rate sampling" where all inputs are updated at one common rate and all outputs are updated at another common rate. State feedback problems under these frameworks are addressed in [24] and pole placement type design methods are given in [25] and [26].

In [3], Meyer studied the multirate LQG problem using standard LQG theory and without simplifying assumptions on the sampling. It is shown that multirate LQG problem can be solved without using periodic system theory or solving periodic riccati equations. Here multirate sampled data LQG problem is translated into an equivalent, modified, single-rate, shift-invariant problem via a lifting isomorphism approach. In [2], A new class of linear shift-varying operators that generalizes the notion of N-periodicity is defined. The stability theory for linear time-invariant plants with periodic digital controllers is considered in [27]. Here it is shown that gain margin can be arbitrarily assigned by suitable choice of sampling time and digital controller.

Examples of direct methods for controller reduction include the work of Gangsaas et al. [28], Hyland and Bernstein [29]. For indirect method good number of references can be found in [6]. A sampled data controller reduction procedure is considered in

[30]. Here weighted balanced truncation is used to reduce the order of the controller designed for multirate sampled system using lifting technique. Anderson and colleague have developed a frequency weighted hankel norm technique for controller reduction [6], [31]. A frequency error bound for the frequency weighted controller order reduction when stability is preserved can be found in [32].

There are now at least three important and also rather popular state space based model reduction techniques, namely, truncation of internally balanced realization [33], [34] Hankel norm optimal approximation [35], [36] and q-covariance equivalent realization (q-cover) [37], [38], [39]. Other approaches employing balancing to achieve controller reduction includes work of Yousuff and Skelton [40] and Davis and Skelton [41]. A general information on controller design is given in [42]. In [43] optimal hankel model reduction for nonminimal systems is discussed. Balancing transformations for unstable nonminimal linear system is given in [44].

A discrete time controller is obtained by finding the zero order hold equivalent of the open loop controller. It is shown in [45] that this may lead to poor performance because it does not seek directly to have the continuous time closed loop closly approximated by the sampled data closed loop. Stability of sampled data feedback systems is discussed in [46].

The stability robustness analysis in face of structured uncertainty for continuous time case had been given by Yedavalli [47], Barmish [48], Zhou and Khargonekar [49], *Holé* [50], Kosmidou [51]. A simple test for determining the stability of a characteristic polynomial whose coefficients vary around some set of nominal values has also been developed.For such type of interval polynomial, Kharitonov showed that stability can be determined from the stability of four fixed polynomials. Extension of Kharitonov theorem to discrete case are given in [52], [53].

In [54], smallest destabilizing perturbations for linear systems(both for continuous and discrete time case) have been derived. A sufficient condition for the stability robustness of linear systems with time varying structured uncertainty is given by Sobel et al. [55]. Here it is shown that the nominal eigenvalues lie to the left of a vertical

line in the complex plane which is determined by a norm involving the structure of the uncertainty and nominal closed loop eigenvector matrix. In [56], element bound for allowed perturbations in system matrices is derived based on discrete lyapunov stability approach. Niu et al. [57], have given bounds for allowed perturbations in system matrices using discrete lyapunov stability approach.

Luo and Johnson [58], have given a saddle point inequality of the closed loop system performance to cope with noise uncertainty, and a sufficient condition for system stability robustness under plant perturbation. These results are extended in [59], where it is shown that the new robust stability sufficient condition may become necessary and sufficient condition of system stability in certain cases.

1.3 Proposed Work In Brief

In the present work closed loop performance of a sampled data system via lifting technique is considered. Based on results given in [1] lifting technique is used to replace the periodically time-varying sampled-data system by a single-rate time-invariant system.

Based on result given in [6] controller order reduction by balanced realization truncation and frequency weighted balanced realization truncation (both for discrete time controller and controller obtained after discretizing the continuous time controller) is considered. Also bounds on maximum perturbations in entries of system matrices are calculated using results given in [59].

The effectiveness of the theory has been shown by designing digital autopilot for launch vehicle.

1.4 Outline Of The Thesis

The Chapter 2, presents fast sampling followed by lifting for the multirate hybrid closed-loop system containing plant and controller. A time-invariant system results. In Chapter 3, Controller order reduction by balanced realization truncation method and frequency weighted balanced truncation method is considered. In case of frequency

weighted balanced truncation method, weighting functions for preserving the closed loop transfer function is presented. A procedure for controller order reduction in both the cases is given. In Chapter 4, stability robustness of the systems with structured time varying uncertainties is considered. Also a practical example of designing digital autopilot for launch vehicle is given. The Chapter 5, concludes the thesis.

Chapter 2

FAST SAMPLING AND LIFTING TECHNIQUE

2.1 Introduction

In this chapter, the fast sampling and lifting technique is discussed. The periodically time-varying sampled-data system is replaced by a single-rate time-invariant system.

The application of the ordinary z-transform to the analysis and synthesis of sampled data system gives information which can be used to evaluate the variables at sampling instants only. However, the behaviour of the system between sampling instants is of considerable importance. For instance, system may have considerable overshoots between sampling instants during the transient period, even though the response at sampling instants has settled completely.

To take into account intersampler behaviour of the system, multirate sampling as described in [30] is used. Then the system becomes periodically,linear,time-varying sampled-data system. Using lifting technique as described in [1], this system is replaced by a single-rate time-invariant system.

Fast sampling and lifting technique is described in Section 2.2. Also, the state model of the lifted plant and lifted controller are presented.

2.2 Lifting Technique

In this section, the problem of replacing the periodically time-varying sampled-data system by a discrete-time, linear, time-invariant system is considered.

Consider a hybrid closed loop system where the plant is continuous-time and the controller is discrete-time (controller may be designed for the discrete time model of the plant, or, may be obtained by descretizing the continuous-time controller).

This closed loop system is shown in Figure 2.1.

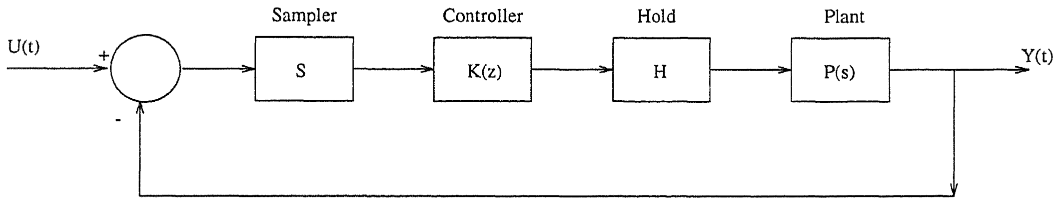

Figure 2.1: The Closed Loop System

P stands for the $p \times m$ continuous-time plant,

K for the $m \times p$ discrete-time controller,

S for the sampler with the sampling period τ, and

H for the hold element,here assumed to be a zero order hold,

m for input of the plant,

p for output of the plant.

This is a periodically time-varying sampled-data system.The aim is to replace the system in Figure 2.1 by a time-invariant system.

To take into account intersample behaviour of the system in Figure 2.1 fast sampling of the system with the sampling time τ/N is introduced. Here τ is the controller sampling time and N is a positive integer, and τ/N is chosen to be a submultiple N of τ.

The fast sampled closed loop system is shown in Figure 2.2

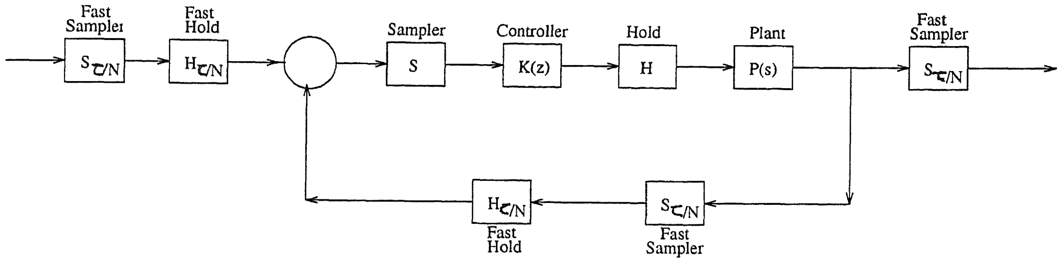

Figure 2.2: The Fast Sampled Closed Loop System

The sampling rate can be chosen based on criteria given in [4] [60]. Normally, τ/N is chosen to be much smaller than the smallest time constant in the scheme of Figure 2.1. So the sampling rate can be chosen to be smaller than the inverse of 5 $\times$ closed loop bandwidth.

The sampled system in Figure 2.2 is a multirate N-periodic discrete-time system. Lifting involves passing from an N-periodic linear $p \times m$ discrete-time system to an equivalent $pN \times mN$ discrete-time linear time- invariant system. Lifting is also known as "Blocking" or "serial to parallel conversion" [42]. Using this technique the multirate system is changed into a single rate system. This technique can be easily understood by looking at Figure 2.3.

In Figure 2.3, the upper system has signals arriving say at 1/secs., while the lower system has one 3 vector arriving every 3 secs.. Likewise, the upper system has one output coming out every second, and the lower system has one 3 vector arriving every 3 secs.. The total information entering and leaving the two systems is identical; it is just presented differently. This shows that a fast system is slowed down and is replaced by an equivalent slow system [42]. If system matrices of a state-variable description of the fast system are known, the corresponding matrices for the slow system can be easily found [1]. This is presented in next section.

Here the equivalence between periodic discrete time system and linear time invariant system is an isomorphism of the systems in the sense that both essential algebraic and analytic properties of the systems are preserved. In particular, the lifted system is

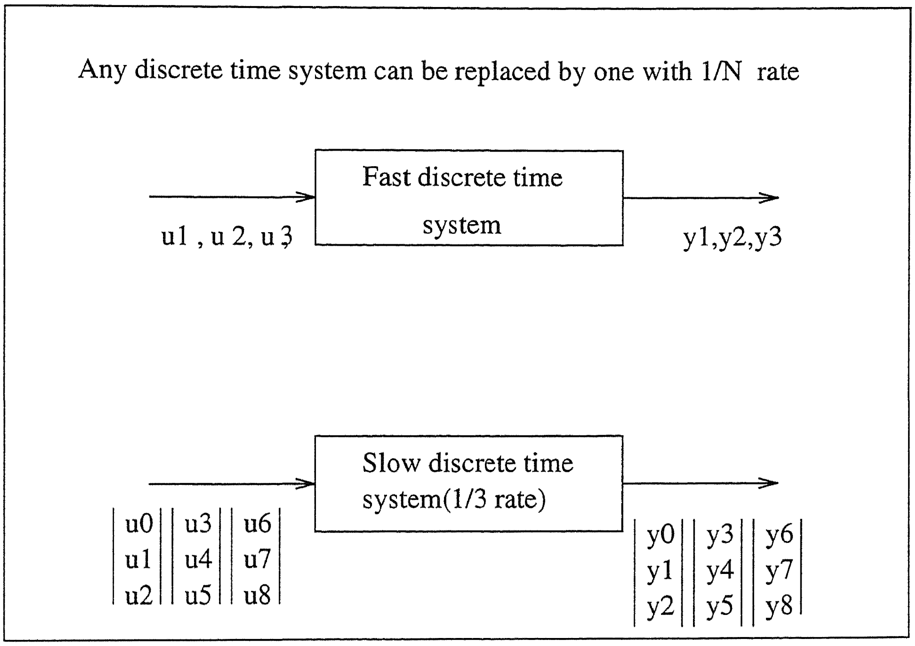

Figure 2.3: Blocking or Lifting on Serial to parallel conversion

stable if and only if the N-periodic system is stable, and in this case the operator norms (associated with regarding the system as an operator mapping square summable input to square summable output) are equal. These results are given in [1], [27].

To obtain a time-invariant system, we lift the system in Figure 2.2. Transfer function of the lifted system is derived next.

2.3 State Models And Transfer Functions

Consider the following continuous-time model of a linear time-invariant plant P(s) :

$$\begin{aligned} \frac{dx_p(t)}{dt} &= A_p\, x_p(t) + B_p\, u(t) \\ z_p(t) &= M_p\, x_p(t) + V_p\, u(t) \\ y_p(t) &= C_p\, x_p(t) + D_p\, u(t) \end{aligned} \tag{2.1}$$

where x_p is the state vector of dimension $(n_p \times 1)$, u is the control vector of dimension $(n_u \times 1)$, y_p is the output vector of dimensions $(n_y \times 1)$, and z_p is the measurement vector of dimensions $(n_z \times 1)$.

The discrete-time approximation of fast sampled system is obtained from (2.1) for a sampling period of (τ/N) secs., and a zero order hold. Where τ is controller sampling time and N is a positive integer.

The discrete time model thus obtained is described by the following equation:

$$\begin{aligned} x_d(k+1) &= A_d\, x_d(k) + B_d\, u(k) \\ z_d(k+1) &= M_d\, x_d(k) + V_d\, u(k) \\ y_d(k) &= C_d\, x_d(k) + D_d\, u(k) \\ where & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.2) \\ A_d &= exp(A_p\tau/N) \\ B_d &= \int_0^{\tau/N} exp(A_p t)dt\, B_p \\ C_d &= C_p \\ D_d &= D_p. \qquad\qquad\qquad\qquad\qquad\qquad (2.3) \end{aligned}$$

The dynamic controller $K_c(t)$ for the continuous-time model of the plant is described by the following equation:

$$\begin{aligned} \frac{dx_c(t)}{dt} &= A_c\, x_c(t) + B_c\, z_p(t) \\ u(t) &= C_c\, x_c(t) + D_c\, z_p(t). \end{aligned} \tag{2.4}$$

where x_c is the controller state vector of dimension $(n_c \times 1)$, u and z_p are the control and measurement vectors,described in the plant model.

The discrete-time controller can be found by descretizing the continuous- time controller given in equation (2.4), or, can be found for the discrete-time model of the plant given in equation (2.1). In both the cases the sampling time is τ secs.

For convenience following notations are used.

LQG_S: controller obtained after descretizing the continuous-time controller,

LQG_D: controller obtained for the discrete-time model of the plant.

The discrete-time controller K(z) is described by the following equation:

$$x(k+1) = A\, x(k) + B\, z_d(k)$$

$$u(k) = C\,x(k) + D\,z_d(k) \tag{2.5}$$

where x is the controller state vector of dimensions $(n \times 1)$, u and z_d are the control and measurement vectors, described in the plant model(2.2).

Now the plant described by equation (2.2) is N-periodic discrete-time system. To replace this system by single-rate, linear, time-invariant system, the system is lifted as described in [1]. Then the state model of the lifted plant $\wp$ is

$$\begin{aligned} X_L(k+1) &= A_L\,X_L(k) + B_L\,U(k) \\ Y_L(k+1) &= C_L\,X_L(k) + D_L\,U(k) \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} A_L &= A_d^N \\ B_L &= [A_d^{N-1}B_d \ \ldots \ A_dB_d \ B_d] \\ C_L &= [C_d^T \ A_d^TC_d^T \ \ldots \ (A_d^T)^{N-1}C_d^T]^T \\ D_L &= \begin{bmatrix} D_d & 0 & \ldots & 0 \\ C_dB_d & D_d & \ldots & 0 \\ C_dA_d^{N-2}B_d & C_dA_d^{N-3}B_d & \ldots & D_d \end{bmatrix}. \end{aligned} \tag{2.7}$$

The realization (2.7) is minimal if (2.2) is minimal.

The lifted controller κ is given as below

$$\kappa(z) = E_1\,K(z)\,E_2 \tag{2.8}$$

where

$$\begin{aligned} E_1 &= (I_m \ I_m \ \ldots \ I_m)^t \ \in \Re_{mN \times m}, \\ E_2 &= (I_p \ 0_p \ \ldots \ 0_p) \ \in \Re_{p \times pN} \end{aligned} \tag{2.9}$$

I_n: $n \times n$ identity matrix.

0_p: $p \times p$ zero matrix.

E_2 in equation (2.9) corresponds to a slow (every τ seconds) sampler, which passes through only the first element of an input vector and is in the off mode when the following N-1 elements of the input vector arrive.

E_1 corresponds to a τ-second zero order hold.

E_1 and E_2 in the single-rate system correspond to interpolator and decimator in the multi-rate system. Introducing

$$\bar{P} = \wp E_1 \tag{2.10}$$

the periodically time varying sampled-data system in Figure 2.1 can be replaced by a time invariant system in Figure 2.4.

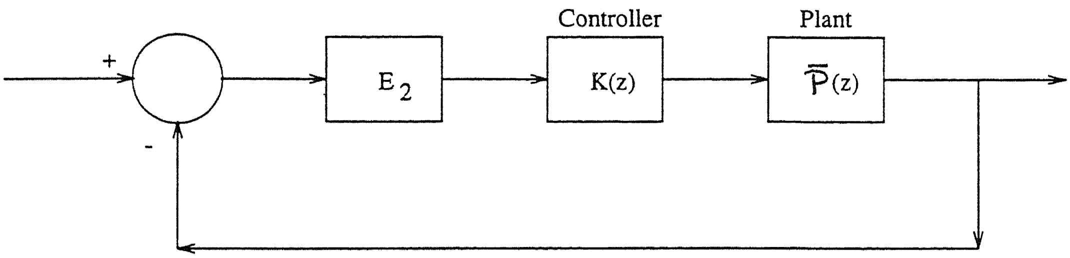

Figure 2.4: The Lifted Closed Loop System

2.4 Conclusions

In this chapter, the lifting technique to replace the periodically time- varying sampled-data system by a single-rate, discrete-time, linear, time-invariant system has been presented. Also, state models of the lifted systems are presented.

....

Chapter 3

CONTROLLER REDUCTION

3.1 Introduction

The controller design methods specially LQG (Linear quadratic gaussian) or H_∞ for physical systems with high order models normally result in a controller of order equal to that of model. Low-order controllers are normally preferred to high-order controllers, given comparable performance: there are fewer things to go wrong in the hardware; their operation is easier to grasp at the conceptual level; that is, one is more likely to be able to identify parts of the controller as achieving certain subgoals of control; and in a discrete-time implementation the computational requirements are less. Therefore, it is desirable to reduce the controller order. Here indirect approach to low order controller design is considered. This uses LQG method to design a high order controller and then this controller is approximated by a low order controller.

In this chapter, the procedure for carrying out this approximation is considered. In Section 3.2, the balanced realization truncation technique is presented and in Section 3.3, the frequency weighted balanced truncation technique for controller order reduction is detailed along with a weighting function for preserving the closed loop transfer function. Also, a balancing transformation matrix is presented.

3.2 Balanced Realization Truncation

let $\bar{P}(z)$ and $K(z)$ be the tranfer function matrices of lifted plant and controller as described in equations (2.10) and (2.5) respectively.

Here $K(z)$ is a stabilizing high order dynamic controller with unity negative feedback. Let $K_r(z)$ be a low order controller, which we are seeking. Closed loop system with controller $K_r(z)$ replacing $K(z)$ is shown in Figure 3.1.

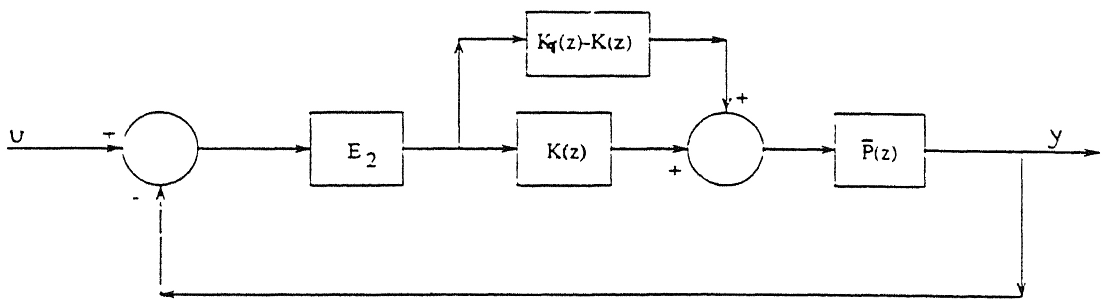

Figure 3.1: Redrawing of closed loop system with controller $K_r(z)$

Then we have the following problem:

Given transfer function matrix $K(z)$ of order n with all poles located inside the unit circle in the z-plane centered at origin, find $K_r(z)$ of order $r < n$ with all poles located inside the unit circle in the z-plane centered at origin, and in addition such that

$$J = \| K(z) - K_r(z) \|_\infty \tag{3.1}$$

is minimum.

The notation $\|A(z)\|_\infty$ means $\sup_{\theta \in [0,2\pi]} \sigma_{max}[A(z)]$. Here, $\sigma_{max}[A(z)]$ denotes the maximum singular value of matrix $A(z)$ and $z = e^{j\theta}$.

Condition (3.1) and the condition that K and K_r have the same number of unstable poles suggest the following procedure for constructing reduced order controller.Let

$$K(z) = K_+(z) + K_-(z). \tag{3.2}$$

where $K_+(z)$ is strictly proper, with all poles outside the unit circle and $K_-(z)$ has all poles inside the unit circle. Now the reduced order controller K_r is represented as

$$K_r(z) = K_+(z) + K_{-r}(z). \tag{3.3}$$

where unstable part of $K(z)$ is copied into $K_r(z)$ and $K_{-r}(z)$ has all poles inside the unit circle. And $K_{-r}(z)$ is chosen to minimize J over all $K_{-r}(z)$ of prescribed degree. Here the requirements of $K(z)$ and $K_r(z)$ to have same number of unstable poles and minimization of criterion given in equation 3.1 are sufficient conditions, not necessary conditions, for stability with the reduced order controllers [60].

To solve this problem, at least in an approximate way, the balanced realization truncation technique [6], [60] is applied to stable part of K.

3.2.1 Procedure For Balanced Realization Truncation

Given an n-th order, linear, time invariant, and asymptotically stable controller with transfer function matrix K(z), where

$$K(z) = C\,(zI - A)^{-1}\,B \tag{3.4}$$

with $\{A, B, C\}$ minimal. Let U, Y be the solution of the following Lyapunov equations

$$\begin{aligned} A\,U\,A^T + B\,B^T &= U \\ A^T\,Y\,A + C^T\,C &= Y \end{aligned} \tag{3.5}$$

where U and Y are the infinite time controlability and observability Gramians. Consider a coordinate transformation matrix T. Let

$$\begin{aligned} A_b &= T^{-1}\,A\,T \\ B_b &= T^{-1}\,B \\ C_b &= C\,T. \end{aligned} \tag{3.6}$$

such that

$$\begin{aligned} A_b\,\Sigma\,A_b^T + B_b\,B_b^T &= \Sigma \\ A_b^T\,\Sigma\,A_b + C_b^T\,C_b &= \Sigma. \end{aligned} \tag{3.7}$$

where

$$\Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n) \qquad \sigma_i \geq \sigma_{i+1}$$

Then the matrix T is called a balancing transformation and $\{A_b, B_b, C_b\}$ is a balanced realization of $\{A, B, C\}$. The matrix Σ is both the controllability and observability Gramians. The Hankel singular value σ_i are obtained as positive square roots of the eigenvalue of UY.

Now, for controller reduction partition the system $\{A_b, B_b, C_b\}$ as

$$A_b = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{array}{l} \}r \\ \end{array}, \quad B_b = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\begin{array}{l} \}r \\ \end{array},$$

$$C_b = [\underbrace{C_1}_{r} \;\; C_2], \quad \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{3.8}$$

and set

$$A_r = A_{11}, \quad B_r = B_1, \quad C_r = C_1. \tag{3.9}$$

Then the reduced order system $\{A_r, B_r, C_r\}$ is a good approximation of system $\{A, B, C\}$ if $\sigma_r \gg \sigma_{r+1}$.
In fact we have the following two properties

- Subsystems $\{A_{ii}, B_i, C_i\}$, i=1,2 are asymptotically stable if $\sigma_r > \sigma_{r+1}$ [34].
- There exists a frequency domain error bound for the balancing approximation [36]

$$\| C(zI - A)^{-1} B - C_r(zI - A_r)^{-1} B_r \|_\infty \leq 2(\sigma_{r+1} + \ldots + \sigma_n) \tag{3.10}$$

Now if $\sigma_{r+2}, \ldots, \sigma_n$ are much smaller than σ_{r+1}, from (3.10) it is clear that balanced realization truncation necessarily comes close to the optimum. Comparisons on practical examples suggests that balanced realization has its own in built frequency shaping which appears to enhance more often than degrade a controller approximation. This reason and its relative simplicity are behind our preferance for the balanced realization approach.

3.3 Frequency Weighted Balanced Realization Technique

In this Section the order of the controller K of the setup in Figure 2.4 using the approach given in [30] is considered.

Any controller reduction procedure should take into account the existence of the plant. Controller reduction should preserve closed-loop stability, and as far as possible, the closed loop performance and closed loop transfer function. This generates frequency weighted approximation problems. The choice of frequency weight is influenced by the choice of criterion used [6]. Here the aim of controller reduction is the preservation of the closed loop transfer function.

The closed loop transfer function matrices with $K(z)$, $K_r(z)$(where $K(z)$, $K_r(z)$ are as described in Section 3.2) are

$$\begin{aligned} \mathcal{T}(z) &= \bar{P}(z)\,K(z)\,E_2\,[\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1} \\ \mathcal{T}_r(z) &= \bar{P}(z)\,K_r(z)\,E_2\,[\,I+\bar{P}(z)\,K_r(z)\,E_2\,]^{-1}. \end{aligned} \tag{3.11}$$

Where $\bar{P}(z)$, E_2 are as described in equation (2.9-2.10). Consider the difference

$$\begin{aligned} &\mathcal{T}(z)-\mathcal{T}_r(z) = \\ &\quad \bar{P}(z)\,K(z)\,E_2\,[\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1}-\bar{P}(z)\,K_r(z)\,E_2\,[\,I+\bar{P}(z)\,K_r(z)\,E_2\,]^{-1}. \end{aligned} \tag{3.12}$$

To a first order approximation in $K(z)-K_r(z)$. there holds

$$\begin{aligned} &\mathcal{T}(z)-\mathcal{T}_r(z) \approx \\ &\quad [\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1}\,\bar{P}(z)\,[\,K(z)-K_r(z)\,]\,E_2\,[\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1}. \end{aligned} \tag{3.13}$$

And this suggests the following approximation problem, Find K_r of nominated degree such that

$K(z)$ and $K_r(z)$ have same number of unstable poles, and

$\|\ W(z)\,[\,K(z)-K_r(z)\,]\,V(z)\ \|_\infty$ is minimized.

$W(z)$ and $V(z)$ obtained from eqn. (3.13) are given below:

$$\begin{aligned} W(z) &= [\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1}\,\bar{P} \\ V(z) &= E_2\,[\,I+\bar{P}(z)\,K(z)\,E_2\,]^{-1}. \end{aligned} \tag{3.14}$$

are the weighting functions.

To solve this problem, at least in an approximate way, the frequency weighted balanced truncation technique [30] is applied to the stable part of $K(z)$ (As mentioned in Section 3.2, the unstable part of $K(z)$ is copied into $K_r(z)$).

Consider asymptotically stable frequency weighting functions (3.14) and associated minimal state variable realizations:

$$\begin{aligned} W(z) &= C_w\,[\,zI - A_w\,]^{-1}\,B_w + D_w \\ V(z) &= C_v\,[\,zI - A_v\,]^{-1}\,B_v + D_v. \end{aligned} \tag{3.15}$$

$W(z)$ and $V(z)$ are stable when the closed loop $\mathcal{T}(z)$ is stable.

Here the basic idea is to change the controllability and observability gramians to reflect the introduction of the frequency weighting. The frequency weighted transfer function $W(z)\,K(z)\,V(z)$ has a representation with the following state space matrices

$$\underline{A} = \begin{bmatrix} A_w & B_w C & B_w D C_v \\ 0 & A & B C_v \\ 0 & 0 & A_v \end{bmatrix} \quad \underline{B} = \begin{bmatrix} B_w D D_v \\ B D_v \\ B_v \end{bmatrix}$$
$$\underline{C} = \begin{bmatrix} C_w & D_w C & D_w D C_v \end{bmatrix}. \tag{3.16}$$

Let

$$\underline{U} = \begin{bmatrix} U_w & U_{12} & U_{13} \\ U_{12}^T & U & U_{23} \\ U_{13}^T & U_{23}^T & U_v \end{bmatrix}. \tag{3.17}$$

and

$$\underline{Y} = \begin{bmatrix} Y_w & Y_{12} & Y_{13} \\ Y_{12}^T & Y & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_v \end{bmatrix}. \tag{3.18}$$

be the solution of the following lyapunov equations

$$\begin{aligned} \underline{A}\,\underline{U}\,\underline{A}^T + \underline{B}\,\underline{B}^T &= \underline{U} \\ \underline{A}^T\,\underline{Y}\,\underline{A} + \underline{C}^T\,\underline{C} &= \underline{Y}. \end{aligned} \tag{3.19}$$

Now U and Y can be regarded as the frequency weighted controllability and observability gramians for the original controller K(z).

Consider a coordinate basis change to $\{A, B, C,\}$ which makes

$$U_{new} = Y_{new} = diag(\mu_1\,,\,\mu_2\,,\,\ldots\,,\,\mu_n\,), \qquad \mu_i \geq \mu_{i+1} \tag{3.20}$$

This new realization $\{A, B, C,\}$ is called a frequency weighted balanced realization. Here the balancing transformation matrix T is found using the algorithm given in [61].

Now the controller reduction is achieved by eliminating the rows and columns of A, B and C corresponding to smallest $(\,\mu_{r+1}\,,\,\mu_{r+2}\,,\,\ldots\,,\,\mu_n\,)$ in $U_{new} = Y_{new}$. This yields $K_r(z)$. This part has been explained in Section 3.2 (see equations (3.8) - (3.9)). Here, unlike in nonweighted case (Section 3.2), a stable $K(z)$ may not yield a stable $K_r(z)$. It should be expected that concentration on stability criterion for minimization can lead to poorly performing controllers, while concentration on other performance measures for minimization could lead to instability [6].

3.3.1 A State Space Representation For $W(z)$ and $V(z)$

A state space representation for $W(z)$ described in equation (3.14) can be found from block diagram shown in Figure 3.2. Let the state vectors associated with lifted plant

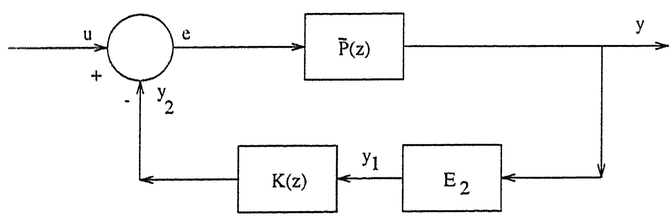

Figure 3.2: Block diagram for obtaining a state space representation for $W(z)$

$\bar{P}(z)$ and controller $K(z)$ be $x_1(k)$ and $x_2(k)$ respectively. State models of lifted plant and controller are given in Section 2.3. Let the input vector be $u(k)$, output vector $y(k)$ and state vector $x(k)$, where $x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$

The state equations for lifted plant can be written as follows

$$\begin{aligned} x_1(k+1) &= A_L\, x_1(k) + B_L\, E_1\, e(k) && (3.21) \\ \textit{and} \quad y(k) &= C_L\, x_1(k) + D_L\, E_1\, e(k) && (3.22) \end{aligned}$$

and the state equations for controller as

$$\begin{aligned} x_2(k+1) &= A\, x_2(k) + B\, y_1(k) && (3.23) \\ \textit{and} \quad y_2(k) &= C\, x_2(k). && (3.24) \end{aligned}$$

Here

$$\begin{aligned} e(k) &= u(k) - y_2(k) && (3.25) \\ \textit{and} \quad y_1(k) &= E_2\, y(k). && (3.26) \end{aligned}$$

Putting eqn. (3.24) in (3.25) and then putting eqns. (3.25) and (3.26) in (3.21), (3.22) and (3.23) respectively, we get

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_L & -B_L\, E_1\, C \\ B\, E_2\, C_L & A - B\, E_2\, D_L\, E_1\, C \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_L\, E_1 \\ B\, E_2\, D_L\, E_1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} C_L & -D_L\, E_1\, C \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + [D_L\, E_1]\, u(k). \tag{3.27}$$

Now A_w, B_w, C_w, D_w obtained from eqn. (3.27) are given below

$$A_w = \begin{bmatrix} A_L & -B_L\, E_1\, C \\ B\, E_2\, C_L & A - B\, E_2\, D_L\, E_1\, C \end{bmatrix}, \quad B_w = \begin{bmatrix} B_L\, E_1 \\ B\, E_2\, D_L\, E_1 \end{bmatrix}$$

$$C_w = \begin{bmatrix} C_L & -D_L\, E_1\, C \end{bmatrix}, \qquad D_w = [D_L\, E_1]. \tag{3.28}$$

Now minimal state variable realization for $W(z)$ is found by deleting the uncontrollable and unobservable states from eqn. (3.27).

A state space representation for $V(z)$ described in eqn. (3.14) can be found from block diagram shown in Figure 3.3. The state equations can be written similarly and

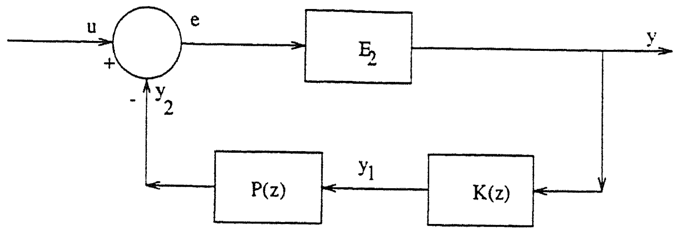

Figure 3.3: Block diagram for obtaining a state space representation for $V(z)$

are solved for $x(k)$. $x(k)$ so obtained is given below

$$\begin{aligned}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} A_L & -B_L\,E_1\,C \\ B\,E_2\,C_L & A - B\,E_2\,D_L\,E_1\,C \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B\,E_2 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} -E_2\,C_L & -E_2\,D_L\,E_1\,C \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + [E_2]\,u(k). \end{aligned} \tag{3.29}$$

Now A_v, B_v, C_v, D_v obtained from eqn. (3.29) are given below

$$\begin{aligned} A_v &= \begin{bmatrix} A_L & -B_L\,E_1\,C \\ B\,E_2\,C_L & A - B\,E_2\,D_L\,E_1\,C \end{bmatrix}, \quad B_v = \begin{bmatrix} 0 \\ B\,E_2 \end{bmatrix} \\ C_v &= \begin{bmatrix} -E_2\,C_L & -E_2\,D_L\,E_1\,C \end{bmatrix}, \qquad D_v = [E_2]. \end{aligned} \tag{3.30}$$

Now minimal state variable realization for $V(z)$ is found by deleting the uncontrollable and unobservable states from eqn. (3.29).

3.4 Construction Of Balancing Transformation

An algorithm [61] is presented below which computes the state space balancing transformation defined in Section 3.2 and Section 3.3.

Step 1 : Compute Cholesky decomposition of Gramians U and Y:

Let R_c and R_o denote the lower triangular Cholesky factors of the Gramians U and Y, i.e.

$$U = R_c\,R_c^T, \quad Y = R_o\,R_o^T.$$

Step 2 : Compute singular value decomposition of the product of the Cholesky factors:

Compute the SVD

$$R_o^T R_c = G\,\Sigma\,S^T. \tag{3.31}$$

$$\Sigma = diag\,(\sigma_1\,, \sigma_2\,, \ldots\,, \sigma_n\,),\;\; \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0.$$

Step 3 : Form the balancing Transformation:

$$\begin{aligned} T &= R_c\,S\,\Sigma^{-1/2}. \\ \text{also}\;\; T^{-1} &= \Sigma^{-1/2}\,G^T\,R_o^T. \end{aligned} \tag{3.32}$$

here

$$T^{-1}\,U\,T^{-T} \;=\; \Sigma. \tag{3.33}$$

and

$$T^T\,Y\,T \;=\; \Sigma. \tag{3.34}$$

From equations (3.31), (3.33) and (3.34) it is clear that the controllability and observability gramians are equal and diagonal.

3.5 Implementation of the Controller Order Reduction Procedure

First, we see how to obtain K for the LQG problem. The continuous time plant is described by equation (2.1). The discrete time model of the plant can be found from equations (2.2-2.3), for a sampling period of τ secs., $N = 1$ and a zero order hold. The continuous time model of the plant is given as follows

$$\begin{aligned} \frac{dx_p(t)}{dt} &= A_p\,x_p(t) + B_p\,u(t) + w_p(t) \\ z_p(t) &= M_p\,x_p(t) + V_p\,u(t) + v_p(t). \end{aligned} \tag{3.35}$$

Where w_p and v_p are assumed to be mutually independent, zero mean, continuous time noise processes with positive definite covariance matrices W_c and V_c, respectively.

The continuous time controller is described by the following equation

$$\frac{dx_c(t)}{dt} \;=\; A_p\,x_c(t) + B_p\,u(t) + H_c\,(\,z_p(t) - M_p\,x_c(t)) \tag{3.36}$$

and control input by

$$u(t) = -F_c x_c(t). \tag{3.37}$$

Here

H_c : Kalman filter gain matrix for continuous time plant

F_c : Feedback gain matrix which is obtained so as to minimize the following quadratic cost function [60]

$$J_c = \int_0^\infty (z_p^T(t) Q_c z_p(t) + u^T(t) R u(t)) dt \tag{3.38}$$

where u and z_p are control and measurement vectors, Q_c is symmetric positive semidefinite matrix and R is positive definite weighting matrix of appropriate dimensions. For the existence and uniqueness of the solution to the LQR we assume standard controllability and observability conditions.

The discrete time model of the plant is given as follows

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d u(k) + w_d \\ z_d(k) &= M_d x_d(k) + V_d u(k) + v_d. \end{aligned} \tag{3.39}$$

Where w_d and v_d are assumed to be mutually independent, zero mean, discrete noise processes with positive definite covariance matrices W_d and V_D, respectively.

The corresponding discrete time controller is described by the following equation

$$x(k+1) = A_d x(k) + B_d u(k) + H_d\,(z_d(k) - M_d x(k)) \tag{3.40}$$

and control input by

$$u(k) = -F_d x(k). \tag{3.41}$$

Here

H_d : Kalman filter gain matrix for discrete time plant

F_d : feedback gain matrix which is obtained so as to minimize the following quadratic cost function [60]

$$J_d \;=\; E \sum_{k=1}^{\infty} (z_d^T(k)\, Q_d\, z_d(k) \;+\; 2 z_d^T(k)\, M_m\, u(k) + u^T(k)\, R_d\, u(k)) \qquad (3.42)$$

where u and z_d are control and measurement vectors, Q_d is symmetric positive semidefinite matrix and R_d is positive definite weighting matrix of appropriate dimensions. For the existence and uniqueness of the solution to the LQR we assume standard controllability and observability conditions. J_d in eqn. (3.42) is found by descretizing J_c in eqn. (3.38) and it in general includes a cross product term M_m in the performance index even though J_c may not contain any. The continuous time controller matrices A_c, B_c, C_c are obtained as given below

$$\begin{aligned} A_c &= A_p - B_p F_c - H_c M_p \\ B_c &= H_c \\ C_c &= -F_c \end{aligned} \qquad (3.43)$$

and the discrete time controller matrices A, B, C are obtained as given below

$$\begin{aligned} A &= A_d - B_d F_d - H_d M_d \\ B &= H_d \\ C &= -F_d. \end{aligned} \qquad (3.44)$$

The LQG_S controller can be found by descretizing the equation (3.43) for a sampling period τ secs. and a zero order hold.

The state feedback gain matrices F_c and F_d and the Kalman filter gain matrices H_c and H_d may be determined by solving the standard LQR and LQG problems for continuous time and discrete time problem respectively. In this thesis these are obtained with the help of software package MATLAB (version 3.12).

3.6 Conclusions

In this Chapter, approximation of high order controller by low order controller has been considered. Controller order reduction by balanced realization truncation and frequency weighted balanced realization truncation has been presented. In case of frequency weighted balanced realization truncation a weighting function for preserving the closed loop transfer function has been presented. Also, a balancing transformation matrix has been presented. Performance criterion for designing the robust optimal controller has been presented.

Chapter 4

NUMERICAL EXAMPLE

4.1 Introduction

In this chapter. a sufficient condition for linear system stability robustness is presented. Also. effectiveness of the theory presented in the previous chapters is seen by designing the digital autopilot for the launch vehicle.

Performance of various reduced order controllers are compared with the existing 9^{th} order LQG controller (with lifting and without lifting the system). The element bounds in the system matrices for the allowed perturbations with the designed controllers are presented. The step responses of various reduced order controllers and 9^{th} order LQG controller are shown and compared with each other.

4.2 A Sufficient Condition For Stability Robustness

Consider a linear discrete-time closed loop system described by

$$x(k+1) = \bar{A}x(k). \tag{4.1}$$

It is assumed that stable nominal system (4.1) is subject to linear time-varying parametric uncertainties in the entries of $\bar{A}$ described by $\Delta\bar{A}(k)$. The closed loop

system matrix is given in equation (B.1). Thus the linear discrete-time closed loop systems with time-varying uncertainties is given by

$$x(k+1) \;=\; (\bar{A} - \Delta\bar{A}(k))\,x(k). \tag{4.2}$$

Let bounds be given on the absolute values of the maximum variations in the element of $\Delta\bar{A}(k)$, i.e.

$$\Delta\bar{A}^{+}_{ij} \;\leq\; max\,(\Delta\bar{A}^{+}_{ij}(k))\;\; \forall k = 0,1,\ldots. \tag{4.3}$$

and $\Delta\bar{A}$ is defined as a matrix with entries $max\,\Delta\bar{A}^{+}_{ij}(k)$. $\Delta\bar{A}(k)$ and $\Delta\bar{A}^{+}(k)$ matrices are given in eqns. (B.2) and (B.3) respectively. Then the structured uncertainties are defined as

$$\{\,\Delta\bar{A}(k)\,:\,\Delta\bar{A}^{+}(k)\,[\leq]\,\Delta\bar{A}\;\;\forall\; k \;=\; 0,1\ldots\,\}. \tag{4.4}$$

Here in $\Delta\bar{A}^{+}(k)\,[\leq]\,\Delta\bar{A}$ '$\leq$' is applied element-by-element to two matrices.
A^{+} : A matrix obtained by replacing the entries of A with their absolute values.
Further, we define

$$\epsilon = max\,(\Delta\bar{A}_{ij})\quad i = 1,\,\ldots\,,n; \quad j = 1,\,\ldots\,,n \tag{4.5}$$

and E is a matrix with entries

$$E_{ij} \;=\; \frac{\Delta\bar{A}_{ij}}{\epsilon}. \tag{4.6}$$

Then the structured uncertainty (4.4) can be rewritten as

$$\{\Delta\bar{A}(k)\,:\,\Delta\bar{A}^{+}(k)\,[\leq]\,\epsilon E \quad \forall\; k \;=0,1\ldots\}. \tag{4.7}$$

Now the stability robustness problem can be stated as :

Given a structured uncertainty matrix E, determine the upper bound on ϵ for maintaining asymptotic stability of the closed loop system (4.2) for all $\Delta\bar{A}(k)$ described by (4.7). The ϵ can be found with the help of theorem given in [59], see also [58].

Now in the following theorem, a sufficient condition for stability robustness is given.

Theorem 4.1 [59]

Suppose the nominal closed loop system described by (4.1) is asymptotically stable, and M is the modal matrix to transform the nominal closed loop system matrix $\bar{A}$ to a Jorden form matrix $\bar{B}$. Then the uncertain closed loop system given by (4.2) is asymptotically stable for all $\Delta\,\bar{A}(k)$ described by (4.7), if

$$\epsilon \;<\; \frac{1}{\pi\,[\,(\,I-\bar{B}^{+}\,)^{-1}\,(\,M^{-1}\,)^{+}\,E\,M^{+}\,]}. \tag{4.8}$$

where

$\pi(A)$: Perron-Frobinius radius, i.e., the maximum eigenvalue of non-negative matrix A.

4.3 Design Of Autopilot For Launch Vehicle

4.3.1 System Model

The launch vehicle has been modeled as 1-input, 3-output, 23^{rd} order continuous-time system. The discrete-time model is obtained from the corresponding continuous-time model for a sampling period of 0.04 secs.,$N = 2$ and a zero order hold. Zeros of the full order continuous-time nominal model as well as those of the two sets of perturbed continuous time model are given in Table A.1. Zeros of the discrete time nominal model and the perturbed models are given in Table A.2.

The reduced order model of the launch vehicle is of the 9^{th} order. Both the continuous-time and discrete-time full order as well as reduced order models are nonminimum. The reduced order continuous-time and discrete-time model are used for the design of the controller. Loop transfer recovery is only partial with the nonmimimum system. Controller order is further reduced using technique presented in Chapter 3. The designed controller is implemented on the full order system.

The reduced order continuous-time model is characterized by matrices in (A.1), (A.2), (A.3), (A.4), (A.5) and (A.6). The discrete-time model is obtained from the corresponding continuous-time model for a sampling period of 0.04 secs., $N = 1$ and a zero order hold using equation (2.2). State weighting matrices Q_c and Q_d are given by (A.7) and (A.10) respectively. Input weighting matrices R and R_d are given by (A.9) and (A.8) respectively Covariance matrices W_c and W_d of the fictitious system noise process injected into the system for the loop recovery are given by (A.11) and (A.12) respectively. Covariance matrices V_c and V_D of the measurement noise process are given by (A.13) and (A.14) respectively. Stability factor are given in (A.15) and (A.16) respectively.

The above mentioned matrices are used for designing the LQG_S (Controller obtained after descretizing the continuous time controller) and LQG_D (controller obtained for the discrete time model of the plant) controllers. The designed controllers should satisfy following performance measures for the closed loop system:

Overshoot less than 6%.

Gammargin ≥ 6 dB.

Phasemargin $\geq 35°$.

Bandwidth approximately equal to or slightly greater than 5 rad/sec..

Feedback should tolerate a delay of 40msec.

The designed controllers must work satisfactorily for the two perturbed models of the launch vehicle.

4.3.2 Robust Optimal Controller Design

For simplicity following notations are used

LQG_{BD} Balanced realization of LQG_D controller.

LQG_{BS} Balanced realization of LQG_S controller.

LQG_{FBD} Frequency weighted balanced realization of LQG_D controller.

LQG_{FBS} Frequency weighted balanced realization of LQG_S controller.

To determine A, B and C matrices for the controller $K(z)$ and A_c, B_c and C_c for the controller $K_c(t)$. Feedback gain matrices F_d and F_c and the Kalman filter gain matrices H_d and H_c are determined as mentioned in Section 3.5. The feedback gain matrices F_d and F_c obtained are given in (A.17) and (A.19) respectively. The Kalman filter gain matrices H_d and H_c obtained are given in (A.18) and (A.20) respectively. From F_d and H_d A, B and C matrices for LQG_D controller are found using equation (3.44). From F_c and H_c A_c, B_c and C_c matrices for $K_c(t)$ controller are found using equation (3.43). Now the LQG_S controller is found by descretizing the $K_c(t)$ controller for a sampling period of 0.04 secs., $N = 1$ and a zero order hold.

With the above mentioned LQG_D controller and LQG_S controller, now balanced realization truncation procedure as described in Section 3.2 and frequency weighted balanced realization truncation procedure as described in Section 3.3 can be applied to further reduce the order of controllers. Here, LQG_D and LQG_S controllers are stable. For balanced realization truncation $K_r(z)$ is chosen to minimize the condition given in equation (3.1). The balancing transformation matrix T is found by carrying out steps presented in Section 3.4. With this T matrix A_b, B_b and C_b are determined using equation (3.6). This new realization $\{A_b, B_b, C_b\}$ is a balanced realization.

For frequency weighted balanced realization, weights $W(z)$ and $V(z)$ are formed as described in equations (3.14-3.15), and U and Y are determined from equations (3.17-3.18). Now the balancing transformation matrix T is found out by carrying out steps presented in Section 3.4. With this T matrix A_b, B_b and C_b are determined using equation (3.6). This new realization $\{A_b, B_b, C_b\}$ is a frequency weighted balanced realization.

The A_b, B_b, C_b, Σ and T matrices obtained for LQG_{BD} controller are given by (A.21), (A.22), (A.23), (A.24) and (A.25) respectively. The A_b, B_b, C_b, Σ and T matrices obtained for LQG_{BS} controller are given by (A.26), (A.27), (A.28), (A.29) and (A.30) respectively. The A_b, B_b, C_b, Σ and T matrices obtained for LQG_{FBS} controller are given by (A.31), (A.32), (A.33), (A.34) and (A.35) respectively. The A_b, B_b, C_b, Σ and T matrices obtained for LQG_{FBD} controller are given by (A.36), (A.37), (A.38), (A.39) and (A.40) respectively.

Now A_r, B_r and C_r for balanced realization truncation and frequency weighted balanced realization truncation can be found as mentioned in equation (3.9). This constitutes the required reduced order controller.

Performance of various reduced order controllers, with and without lifting the system are considered next. Lifting of plant and the closed loop system is done as mentioned in Section 2.2. Here $\tau = 0.04$ secs. and $N = 2$ is taken. So the plant is fast sampled at 0.02 secs. and the controller is sampled at 0.04 secs. This constitutes a multi rate problem. Using lifting technique this multirate problem is replaced by a single-rate problem.The sampling frequency (314.16 rad/sec.) is much higher than the closed loop bandwidth (5 rad/sec.), so antialiasing filter is not needed in our case. Transfer function of the lifted plant is found using equations (2.6-2.7).

Table 4.1 compares performance of the LQG_{BD}, LQG_{BS}, LQG_{FBS} and LQG_{FBD} controllers for different controller order. Here fast sampling of the plant is taken into account. As can be seen from Table 4.1 4^{th} order LQG_{BD} controller, 3^{rd} order LQG_{BS} controller, 7^{th} order LQG_{FBD} controller and 2^{nd} order LQG_{FBS} controller results in lowest % overshoot for the respective controllers. Also, % overshoot in case of LQG_{BD} and LQG_{FBD} controllers is less than in case of the LQG_{BS} and LQG_{FBS} controllers for different controller order. Also, it is seen that performance of 2^{nd} order LQG_{FBS} controller is better than other 2^{nd} order controllers presented. This may be true for the numerical example presented but in general this may not be true. Also, it is seen that performance of above

controller order	Maximum overshoot			
	LQG_{BD} controller	LQG_{BS} controller	LQG_{FBD} controller	LQG_{BDS} controller
9	2.6	7.58	2.6	7.6
8	2.6	7.58	2.6	7.6
7	2.6	7.58	2.6	7.58
6	1	7.7	4.56	8.87
5	1.8	5.9	4.25	6.69
4	2	9.11	21.6	8.3
3	sluggish	4.5	sluggish	12.84
2	oscillatory	8.10	sluggish	2.05
1	sluggish	80	sluggish	13.65

Table 4.1: % overshoot with lifting

mentioned reduced order controllers are better than original 9^{th} order LQG_D and LQG_S controllers. This confirms the findings of other researchers that in many circumstances reduced order controller performs better than high order controller.

Table 4.2 compares performance of LQG_{BD}. LQG_{BS}. LQG_{FBD}, LQG_{FBS} controllers for different controller order and without lifting. As can be seen from Table 4.1 and Table 4.2, lifting results in better performance (less overshoot) . Here with multirate sampling overshoots between sampling instants is reduced considerably compared to single rate sampling. It was seen that further lifting (incresing the sampling rate for the plant) did not improve the performance.

The LQG_D and LQG_S controllers are designed for the reduced order model of order 9, but are implemented on the full order model of order 23 of the launch vehicle. The step responses of the state x_1 (pitch angle) for the 4^{th} order LQG_{BD}, 3^{rd} order LQG_{BS}, 7^{th} order LQG_{FBD} and 2^{nd} order LQG_{FBS} controllers for the

controller order	Maximum overshoot LQG_{BD} controller	LQG_{BS} controller	LQG_{FBD} controller	LQG_{BDS} controller
9	2.65	7.65	2.65	7.65
8	2.62	7.65	2.64	7.65
7	3.32	7.66	2.61	7.65
6	1.3	7.7	4.7	8.95
5	1.8	6.41	4.4	6.9
4	2	9.11	21.75	8.4
3	sluggish	4.92	sluggish	12.95
2	sluggish	8.32	sluggish	2.15
1	sluggish	82	sluggish	13.8

Table 4.2: % overshoot without lifting

full order system are compared with those of the 9^{th} order LQG_D and LQG_S controllers for the nominal model and two sets of perturbed models of the launch vehicle in Figures 4.1 – 4.6. Here step responses for 7^{th} order LQG_{FBD} controller are shown. As can be seen from Table 4.1 controller order can be reduced to 5^{th} order if desired.

The step response of state x_1 for 4^{th} order LQG_{BD} and 7^{th} order LQG_{FBD} controllers for the nominal model (Figure 4.1) shows reduction in maximum overshoot compared to 9^{th} order LQG_D controller. The step response of state x_1 for 3^{rd} order LQG_{BS} and 2^{nd} order LQG_{FBS} controllers for the nominal model (Figure 4.2) shows reduction in maximum overshoot compared to 9^{th} order LQG_S controller. Comparing Figure 4.1 and Figure 4.2, it is found that rise time is less in case of the LQG_S, LQG_{BS} and LQG_{FBS} controllers compared to the LQG_D, LQG_{BD} and LQG_{FBD} controllers.

Controller	Bandwidth (rad/sec.)	Gain margin (dB) with lifting	Gain margin (dB) without lifting	Phase margin (in deg.) with lifting	Phase margin (in deg.) without lifting
$9^{th}order\ LQG_D$	5.05	24.08	23.64	50.26	46.56
$9^{th}order\ LQG_S$	6.02	14.06	12.03	49.65	44.49
$4^{th}order\ LQG_{BD}$	5.12	24.31	23.94	51.50	47.71
$3^{rd}order\ LQG_{BS}$	6.08	14.09	12.02	49.91	44.79
$7^{th}order\ LQG_{FBD}$	5.07	24.09	23.69	51.35	47.51
$2^{nd}order\ LQG_{FBS}$	6.12	15.59	14.31	50.07	44.98

Table 4.3: Bandwidth. Gain margin and phase margin with different controllers

Comparing the step responses of the state x_1 for 4^{th} order LQG_{BD}, 3^{rd} order LQG_{BS}. 7^{th} order LQG_{FBD} and 2^{nd} order LQG_{FBS} controllers for the perturbed system models set I and set II. with that of 9^{th} order LQG_D and LQG_S controllers shown in Figures 4.3 – 4.6. it is observed that the peak overshoot and undershoot are substantially reduced in case of 4^{th} order LQG_{BD}. 3^{rd} order LQG_{BS}. 7^{th} order LQG_{FBD} and 2^{nd} order LQG_{FBS} controllers. From Figures 4.3-4.6 it is seen that settling time is smallest in case of the 2^{nd} order LQG_{FBS} controller. From Figure 4.5 it is seen that settling time is large in case of 4^{th} order LQG_{BD} controller though there is reduction in maximum % overshoot. Taking all the factors (like, % overshoot. rise time. settling time and controller order) into account it can be concluded that performance of 2^{nd} order LQG_{FBS} controller is better than any other controller in two different representation presented for any order.

Two other measures of comparison come from the gain margin and phase margin. Gain margin and phase margin are calculated with and without lifting the system. The results obtained are summerized in tabular form and is presented in Table 4.3.

As can be seen from Table 4.3 gain margin and phase margin are more for 9^{th} order LQG_D, 4^{th} order LQG_{BD} and 7^{th} order LQG_{FBD} controllers compared to 9^{th} order LQG_S, 3^{rd} order LQG_{BS} and 2^{nd} order LQG_{FBS} controllers. Gain margin and phase margin are more with lifting the system for the designed controllers. Also there is small variation in gain margin and phase margin for high order LQG_D and LQG_S controllers and reduced order LQG_D and LQG_S controllers in balanced realization and frequency weighted balanced realization. A time delay of 40 msec. was put in the feedback and the closed loop system was found to be stable. Also bandwidth was calculated with the designed controller and the results obtained are given in Table 4.3.Bandwidth for 2^{nd} order LQG_{FBS} controller is found from Bode magnitude plot shown in Figure 4.7. It can be seen from Figures 4.3-4.6 that the designed controllers work satisfactorily for the two perturbed models of the launch vehicle. From the above discussions it can be concluded that the designed controllers satisfy all the performance measures that have been specified for the closed loop system.

Bounds on allowed perturbations in system matrices are determined as mentioned in Section 4.1. Bounds are calculated with and without lifting the plant. The results obtained are summerized in tabular form and is presented in Table 4.4.

As can be seen from Table 4.4 with lifting, closed loop system is asymptotically stable with more perturbations in system matrices than without lifting. This clearly shows that with lifting, stability robustness to plant parameter perturbation is achieved. Bounds on allowed perturbations in system matrices obtained are compared with the actual perturbations in each entries of the system matrices (this is determined from two sets of perturbed models of the launch vehicle).The actual maximum perturbations in each entries of the closed loop system matrix (containing plant and the controller) for the two sets of perturbed models are as follows

maximum perturbation for the perturbed system model I : 1.95×10^{-5}.

maximum perturbation for the perturbed system model II : 2.60×10^{-5}.

Controller	ϵ with lifting	ϵ without lifting
$9^{th}order\ LQG_D$	1.019×10^{-4}	2.697×10^{-5}
$9^{th}order\ LQG_S$	1.140×10^{-4}	2.741×10^{-5}
$4^{th}order\ LQG_{BD}$	1.093×10^{-4}	2.673×10^{-5}
$3^{rd}order\ LQG_{BS}$	1.138×10^{-4}	2.916×10^{-5}
$7^{th}order\ LQG_{FBD}$	1.119×10^{-4}	3.250×10^{-5}
$5^{th}order\ LQG_{FBD}$	1.009×10^{-4}	2.984×10^{-5}
$2^{nd}order\ LQG_{FBS}$	1.102×10^{-4}	3.105×10^{-5}

Table 4.4: Perturbation bound with different controllers

By comparison it was seen that the bounds obtained include the actual perturbations in system matrices. Also it is seen from Table 4.4 that 7^{th} order LQG_{FBD} controller accomodates larger perturbation in system matrices than 5^{th} order LQG_{FBD} controller. This shows that the controller which gives lowest % overshoot can accomodate larger perturbation in system matrices.

4.4 Conclusions

In this chapter. A sufficient condition for stability robustness has been presented. Also, robust optimal controllers have been designed for the launch vehicle. The design of the controllers have been done using the reduced order continuous time model and discrete time model of the launch vehicle. Controller order reduction by balanced realization truncation and frequency weighted balanced realization truncation method have been presented. The various reduced order controllers have been implemented on the full order model.

Bounds for the allowed perturbations in system matrices and step responses of the various reduced order controllers have been found out and compared with those of the 9^{th} order LQG_D and LQG_S controllers.

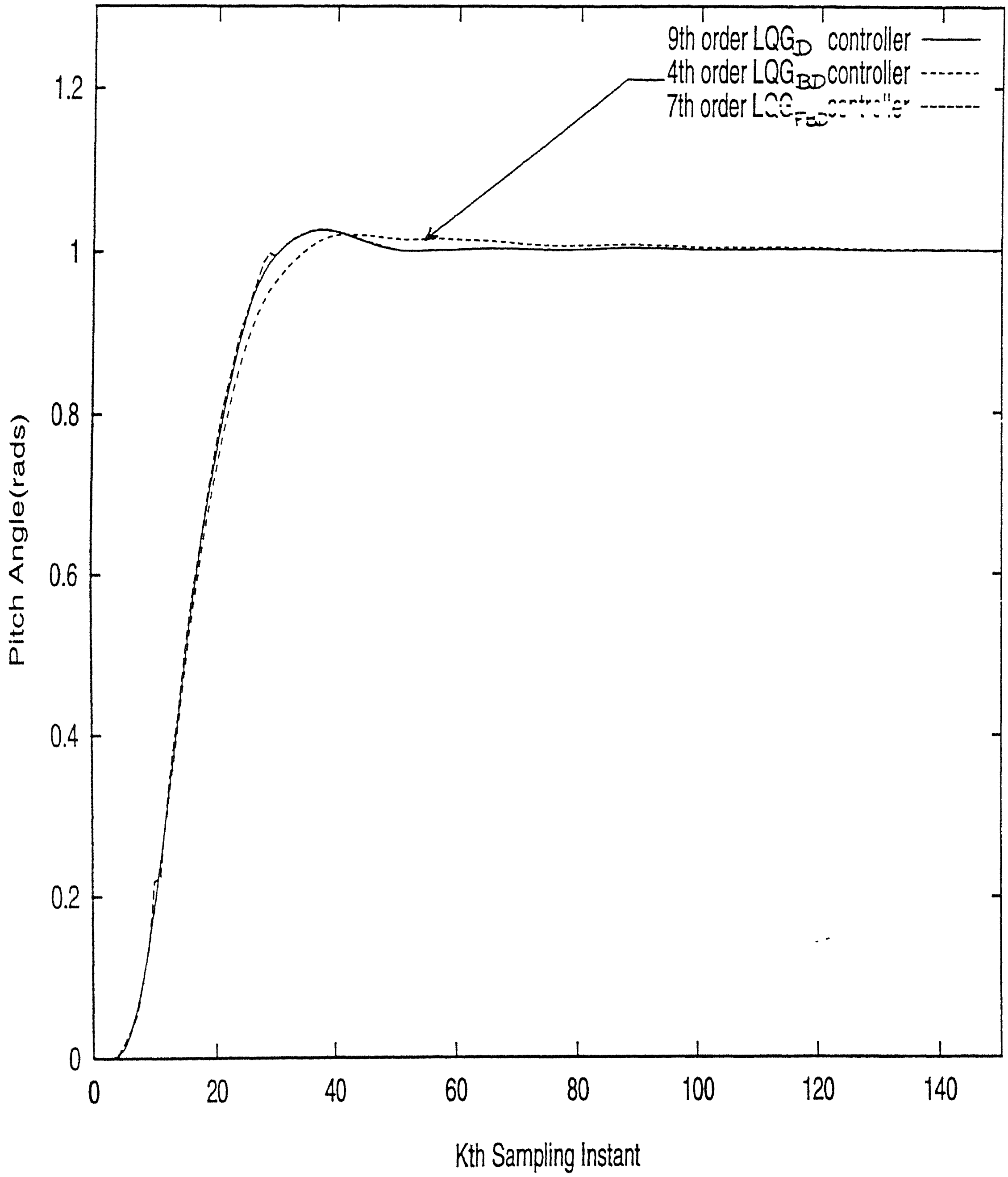

Figure 4.1: Step-Responses of the 9^{th} order LQG_D, 4^{th} order LQG_{BD} and 7^{th} order LQG_{FBD} controllers on full-order(23rd) nominal system.

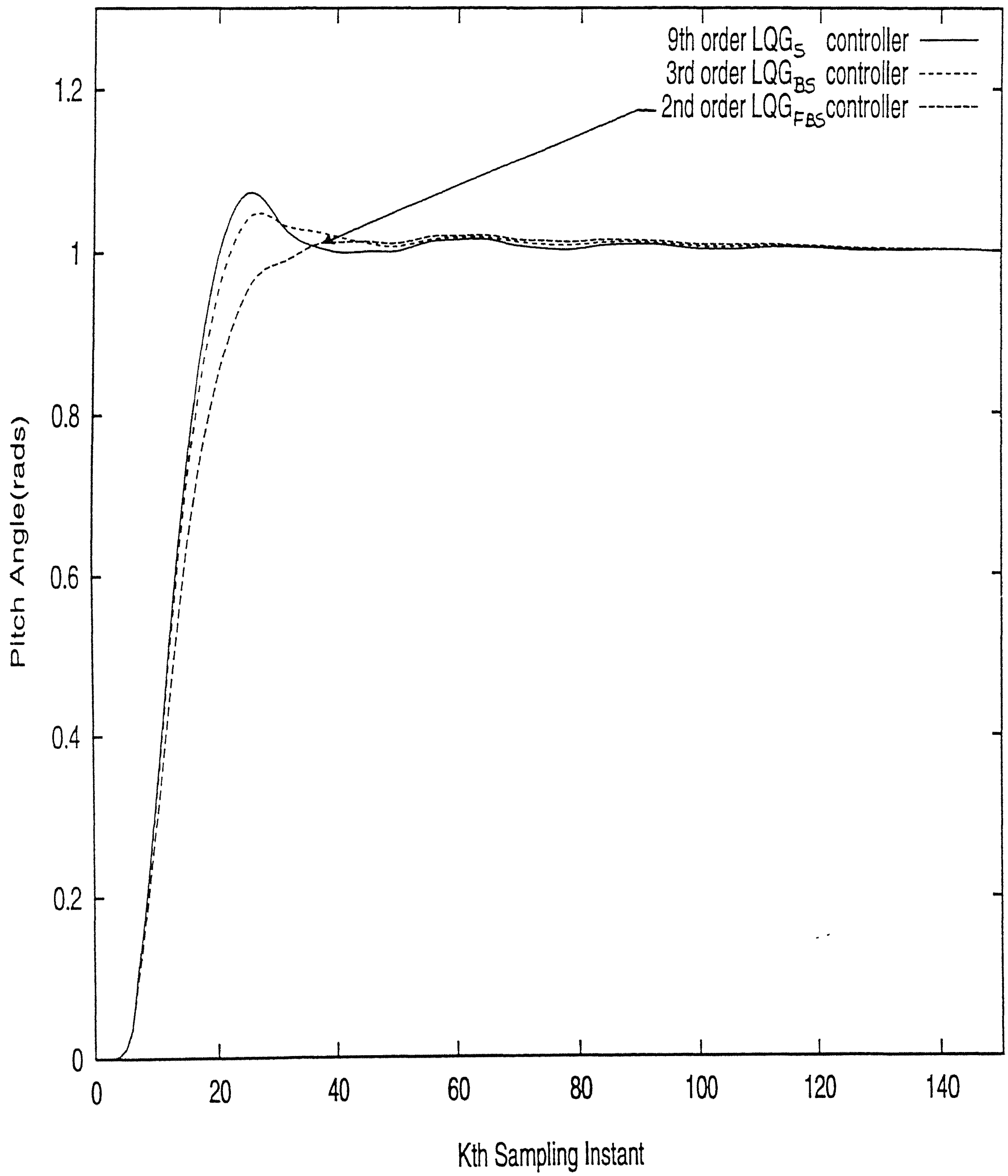

Figure 4.2: Step-Responses of the 9^{th} order LQG_S, 3^{rd} order LQG_{BS} and 2^{nd} order LQG_{FBS} controllers on full-order(23rd) nominal system.

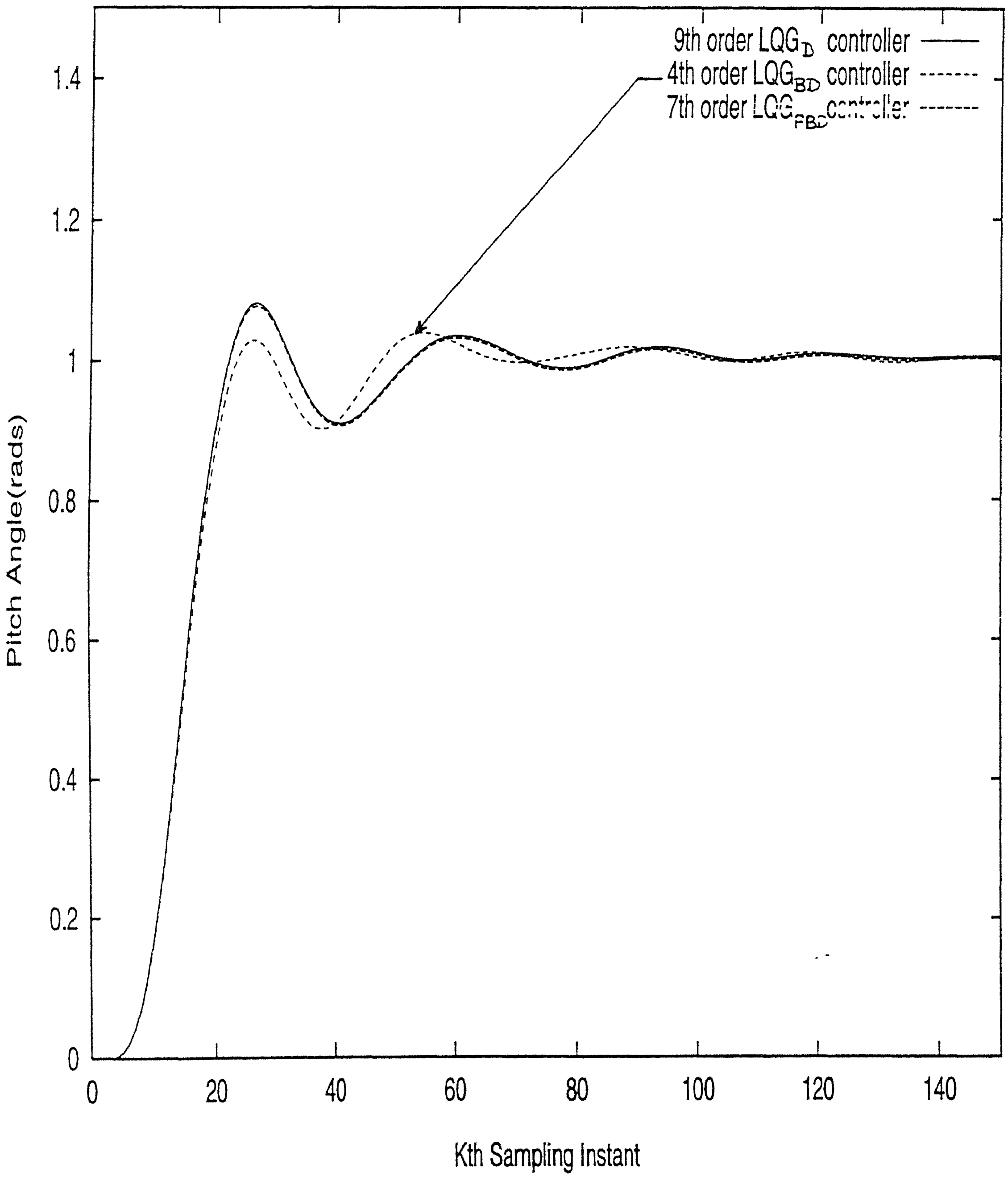

Figure 4.3: Step-Responses of the 9^{th} order LQG_D, 4^{th} order LQG_{BD} and 7^{th} order LQG_{FBD} controllers on full-order(23rd) perturbed system set-I.

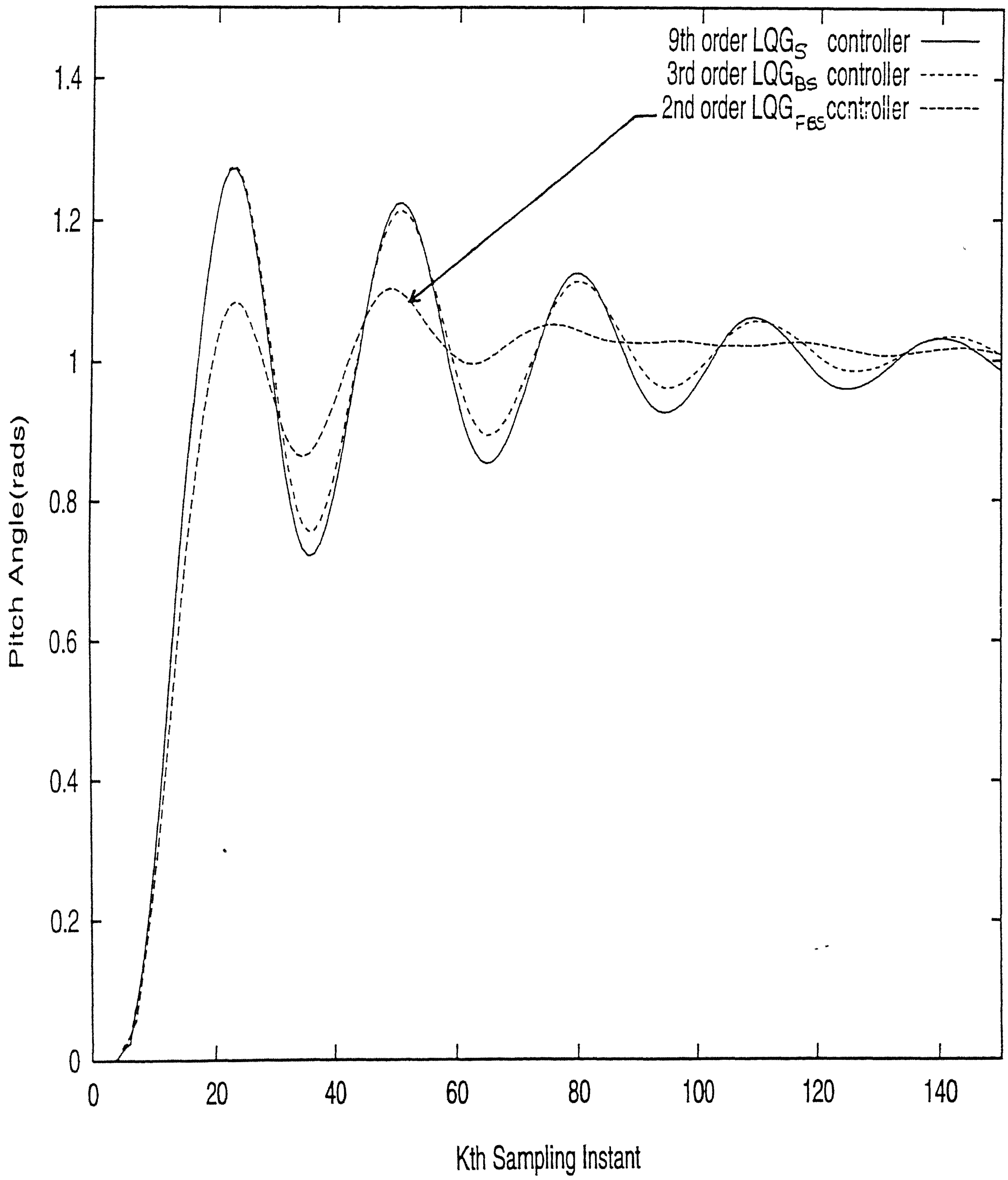

Figure 4.4: Step-Responses of the 9^{th} order LQG_S, 3^{rd} order LQG_{BS} and 2^{nd} order LQG_{FBS} controllers on full-order(23rd) perturbed system set-I.

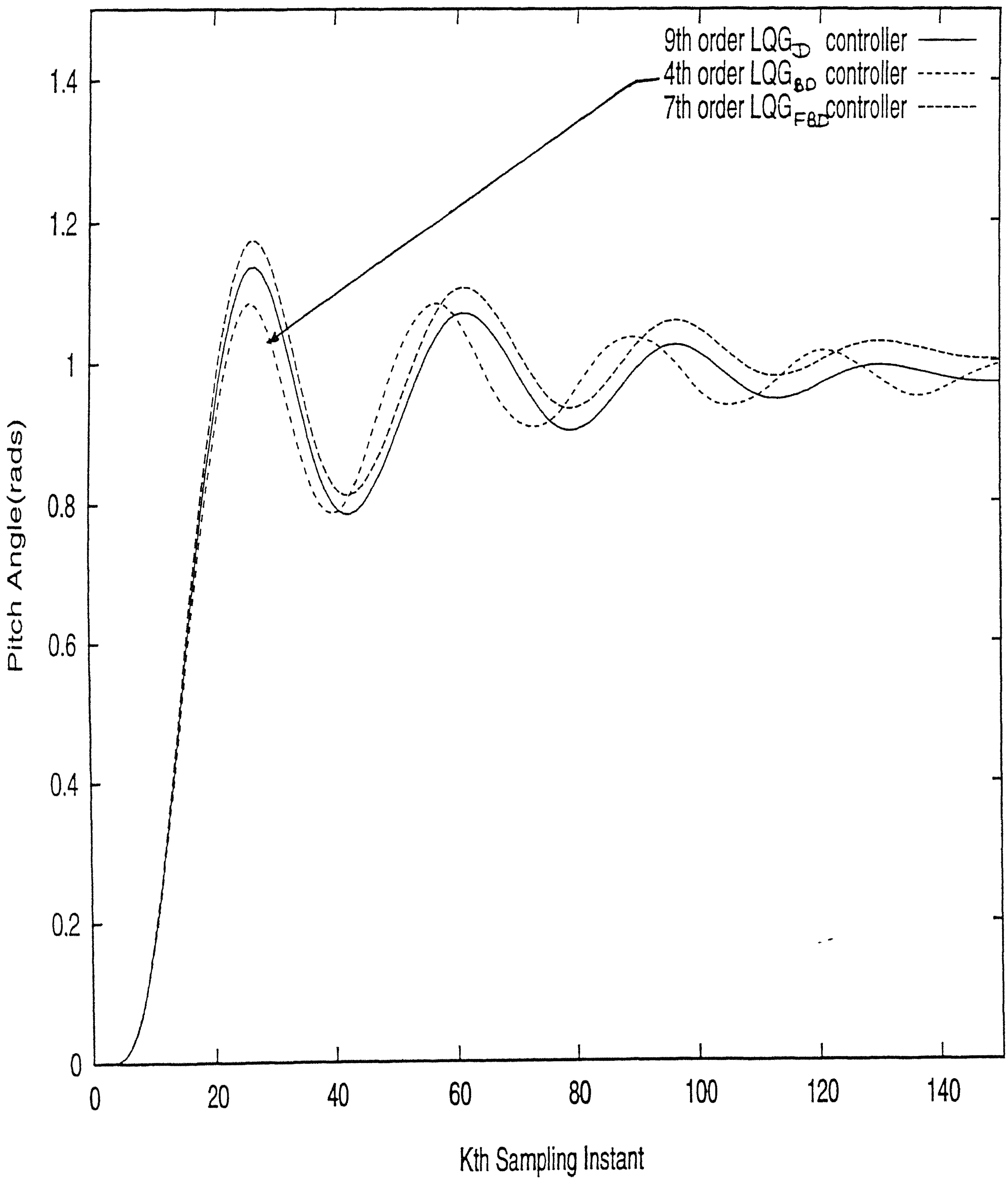

Figure 4.5: Step-Responses of the 9^{th} order LQG_D, 4^{th} order LQG_{BD} and 7^{th} order LQG_{FBD} controllers on full-order(23rd) perturbed system set-II.

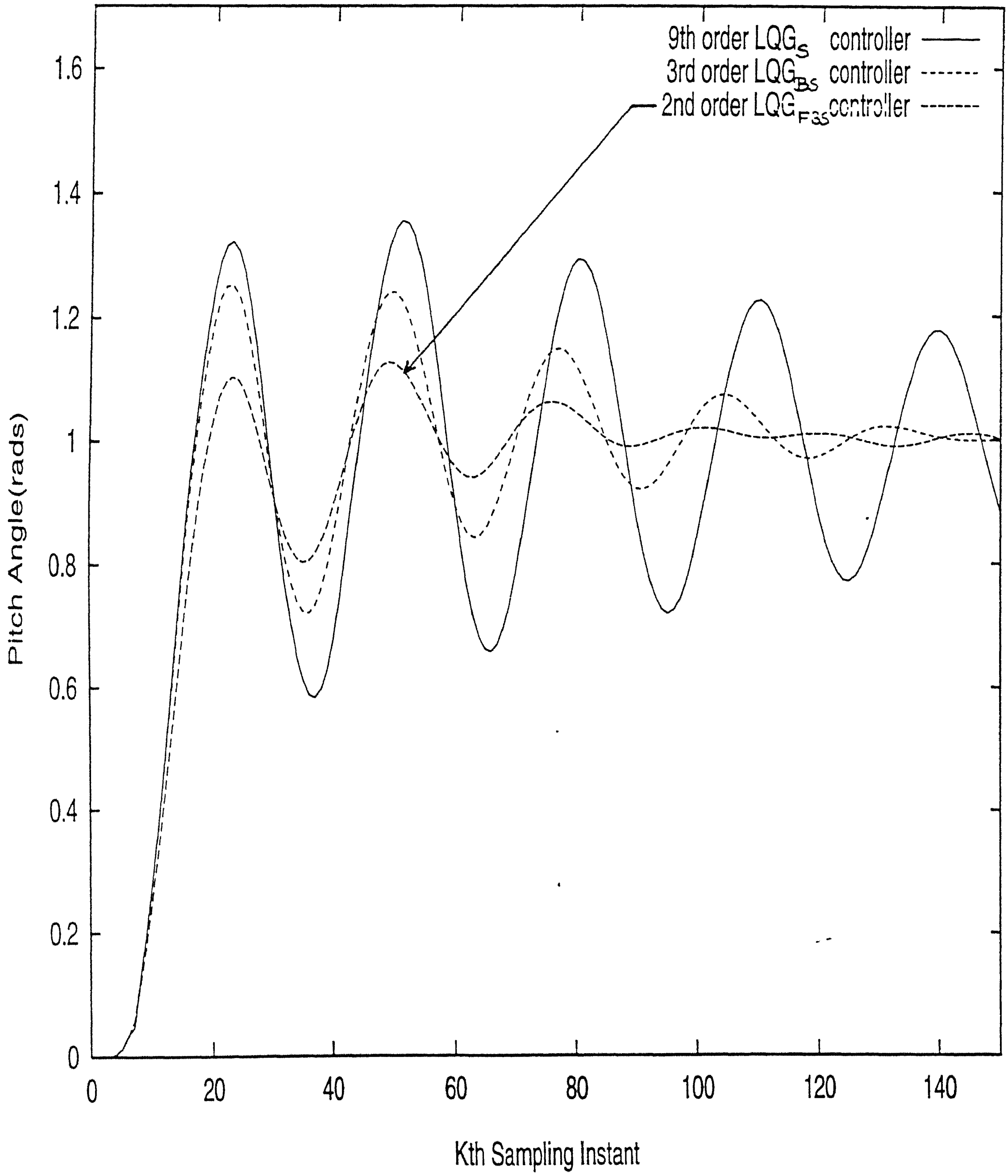

Figure 4.6: Step-Responses of the 9^{th} order LQG_S, 3^{rd} order LQG_{BS} and 2^{nd} order LQG_{FBS} controllers on full-order(23rd) perturbed system set-II.

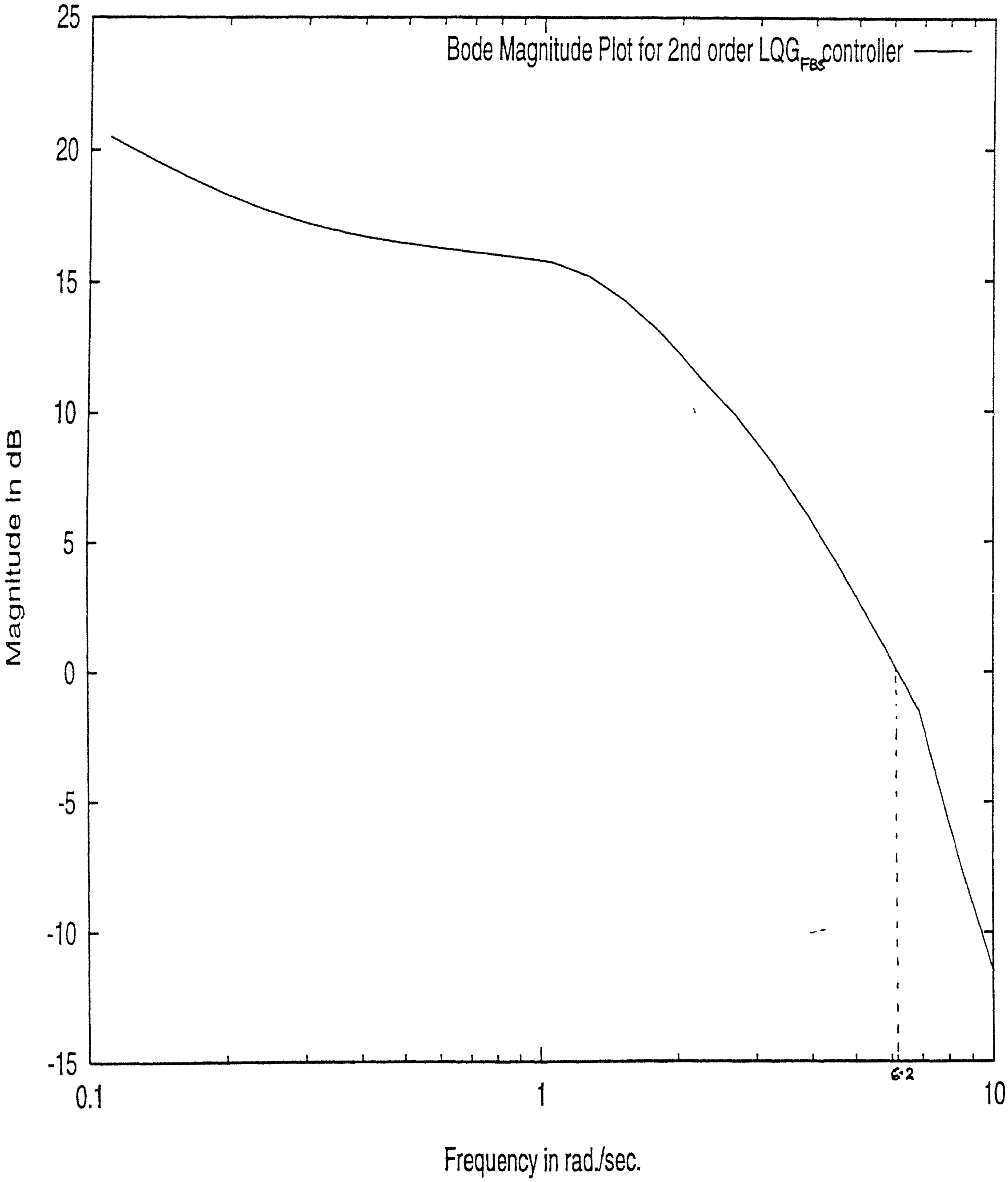

Figure 4.7: Bode magnitude plot with $2nd$ order LQG_{FBS} controller

Chapter 5

CONCLUSIONS

In this thesis. performance of a hybrid closed loop system with and without lifting is considered. Here. the aim is to design a robust optimal controller. Two cases has been considered:

Controller designed for discrete-time model of the plant.

Controller obtained by descretizing the continuous-time controller.

The controllers has been designed using the reduced order model of the launch vehicle and is implemented on the full order model of the launch vehicle. Controller order is further reduced by balanced realization truncation and frequency weighted balanced truncation methods. A comparison has been made of the performance of the system with lifting the plant and without lifting the plant. Also. a comparison has been made of the performance of the various reduced order controllers with the previously designed controller for the same system.

The comparison clearly indicates the benefit of lifting the system. It was seen that increasing the sampling rate further did not improve the performance.It was seen that % overshoot for the closed loop system with LQG_{BD} and LQG_{FBD} controllers is less than with LQG_{BS} and LQG_{FBS} controllers for different controller order. But taking all the factor (like, % overshoot, rise time, settling time and controller order) into account it has been found that performance of the given hybrid closed loop system with the designed 2^{nd} order controller in frequency weighted balanced realization is better than any other controller in two different representations presented for any order.

The element bounds in the system matrices for the allowed perturbations with the designed controllers has been obtained. It was seen that with lifting closed loop system is asymptotically stable with more perturbations in system matrices than without lifting.

Appendix A

In this Appendix the data for the reduced order system, the feedback gain matrices and the Kalman filter gain matrices for continuous and discrete time systems along with balanced controller matrices are given.

Zeros of the Nominal System Model	Zeros of the Perturbed Model Set I	Zeros of the Perturbed Model Set II
-1.5592 $\times 10^6$	-0.3022 + 0.5234i $\times 10^6$	0.2875 + 0. 4978i $\times 10^6$
0.7794 + 1.3502i $\times 10^6$	-0.3022 - 0.5234i $\times 10^6$	0.2875 - 0.4978i $\times 10^6$
0.7794 - 1.3502i $\times 10^6$	0.6044 $\times 10^6$	-0.5747 $\times 10^6$
-41.3420 +29.5080i	51.6977 +19.0658i	70.8324
-41.3420 -29.5080i	51.6977 -19.0658i	-56.3889
42.5823 +25.6410i	-49.1475 +26.9139i	-0.4879 +48.0641i
42.5823 -25.6410i	-49.1475 -26.9139i	-0.4879 -48.0641i
-0.4864 +47.9400i	-0.4856 +47.8637i	-39.0661
-0.4864 -47.9400i	-0.4856 -47.8637i	29.0544
-0.4221 +38.3530i	-0.4228 +38.3479i	-0.3364 +30.6809i
-0.4221 -38.3530i	-0.4228 -38.3479i	-0.3364 -30.6809i
13.3361	13.6440	12.6686
-11.5938	-11.8496	-10.6272
-1.0904 +10.8832i	-1.0882 +10.8812i	-1.0894 +10.8840i
-1.0904 -10.8832i	-1.0882 -10.8812i	-1.0894 -10.8840i
-1.0174 + 9.9535i	-0.9952 + 9.9518i	-0.9962 + 9.9542i
-1.0174 - 9.9535i	-0.9952 - 9.9518i	-0.9962 - 9.9542i
-0.5891 + 5.9631i	-0.5896 + 5.9586i	-0.5906 + 5.9517i
-0.5891 - 5.9631i	-0.5896 - 5.9586i	-0.5906 - 5.9517i
-0.4531 + 4.5734i	-0.4531 + 4.5715i	-0.4532 + 4.5667i
-0.4531 - 4.5734i	-0.4531 - 4.5715i	-0.4532 - 4.5667i
-0.0426	-0.0640	-0.0617

Table A.1: Zeros of the continuous time full order system model

Zeros of the Nominal System Model	Zeros of the Perturbed Model Set I	Zeros of the Perturbed Model Set II
-6.0904	-6.0066	-5.9702
2.0424 + 1.1486i	2.6081 + 1.0443i	4.1051
2.0424 - 1.1486i	2.6081 - 1.0443i	-0.8801
-0.8542	-0.8781	1.7881
0.5689 + 0.8106i	0.5702 + 0.8097i	0.5669 + 0.8120i
0.5689 - 0.8106i	0.5702 - 0.8097i	0.5669 - 0.8120i
0.7139 + 0.6882i	0.7140 + 0.6881i	0.8121 + 0.5720i
0.7139 - 0.6882i	0.7140 - 0.6881i	0.8121 - 0.5720i
1.3071	1.3124	1.2853
0.3634 + 0.2434i	1.1354	1.1415 + 0.0917i
0.3634 - 0.2434i	1.0764 + 0.1693i	1.1415 - 0.0917i.
1.1058 + 0.1015i	1.0764 - 0.1693i	1.0471 + 0.2369i
1.1058 - 0.1015i	0.9827 + 0.2459i	1.0471 - 0.2369i
1.0135 − 0.2209i	0.9827 - 0.2459i	0.9349 + 0.2598i
1.0135 - 0.2209i	0.9060 + 0.2176i	0.9349 - 0.2598i
0.9354 + 0.2326i	0.9060 - 0.2176i	0.8616 + 0.1760i
0.9354 - 0.2326i	0.8556 + 0.0997i	0.8616 - 0.1760i
0.8936 + 0.1503i	0.8556 - 0.0997i	0.8044 + 0.0431i
0.8936 - 0.1503i	0.7809	0.8044 - 0.0431i
0.8653	0.3213 + 0.1918i	0.4578
0.7967	0.3213 - 0.1918i	0.3240
-0.1102	-0.1179	-0.1198

Table A.2: Zeros of the discrete time full order system model (sampling period =0.02 secs.)

System matrices are given in the following equations

$$A_p = \left[\begin{array}{ccccccccc}
1.0655 & 0.0000 & 0.0004 & -0.0001 & -0.0004 & -0.0005 & 0.0001 & -0.0007 & -0.0008 \\
0.0002 & 0.0157 & -0.0005 & 0.0003 & -0.0005 & 0.0015 & -0.0001 & 0.0015 & -0.0034 \\
-0.0002 & 0.0000 & -0.2842 & 15.6628 & -0.0008 & 0.0001 & -0.0001 & 0.0001 & 0.0001 \\
-0.0004 & 0.0000 & -15.6626 & -0.2844 & -0.0001 & 0.0010 & 0.0000 & 0.0002 & -0.0028 \\
0.0003 & 0.0002 & 0.0009 & -0.0020 & -0.4107 & 32.9150 & -0.0021 & -0.0049 & 0.0089 \\
0.0006 & -0.0002 & 0.0046 & 0.0003 & -32.9152 & -0.4112 & -0.0009 & -0.0046 & -0.0063 \\
0.0000 & 0.0000 & -0.0001 & 0.0001 & -0.0009 & 0.0006 & -1.0064 & 0.0008 & -0.0055 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & -0.0002 & 0.0000 & -19.7922 & 20.1921 \\
0.0000 & 0.00000 & 0.0000 & 0.0000 & 0.0003 & 0.0000 & 0.0000 & -20.1918 & -19.7919
\end{array}\right] \tag{A.1}$$

Input matrix

$$B_p = \begin{bmatrix} 14.5943 & -1.2734 & 34.5310 & 13.7632 & 20.9145 & 17.5734 & 15.0813 \\ & 39.5588 & 40.3562 \end{bmatrix} \tag{A.2}$$

Output matrix

$$M_p = \begin{bmatrix} 1.1402 & 1.4732 & 0.0962 & 0.1866 & -0.0854 & -0.0408 & -1.0777 & -0.0095 & -0.0380 \\ 1.1399 & 0.0231 & 0.1062 & -0.0713 & -0.5909 & 1.1713 & 1.0212 & -0.4289 & -0.6922 \\ 22.7444 & 0.4548 & 93.7207 & 200.6986 & 64.2873 & 31.1656 & -22.6467 & 127.7070 & -306.0710 \end{bmatrix} \tag{A.3}$$

$$C_p = \begin{bmatrix} 1.1402 & 1.4732 & 0.0962 & 0.1866 & -0.0854 & -0.0408 & -1.0777 \\ & -0.0095 & -0.0380 \end{bmatrix} \tag{A.4}$$

$$V_p = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{A.5}$$

$$D_p = [\,0\,] \tag{A.6}$$

State weighting matrix for continuous time plant

$$Q_c = C_p^T \times C_p \tag{A.7}$$

$$M_m = \begin{bmatrix} -0.2987 \\ -0.4202 \\ 0.0011 \\ -0.0363 \\ -0.0207 \\ 0.0354 \\ 0.2681 \\ 0.0063 \\ 0.0057 \end{bmatrix} * (10^{-4}), \qquad R_d = 0.2000. \tag{A.8}$$

Input weighing matrix for continuous time plant

$$R = 5. \tag{A.9}$$

State weighing matrix for discrete time plant

$$Q_d = \begin{bmatrix} 0.0543 & 0.0687 & 0.0015 & 0.0094 & -0.0018 & -0.0036 & -0.0492 & 0.0001 & -0.0012 \\ 0.0687 & 0.0869 & 0.0020 & 0.0119 & -0.0023 & -0.0046 & -0.0623 & 0.0002 & -0.0015 \\ 0.0015 & 0.0020 & 0.0001 & 0.0003 & -0.0001 & -0.0001 & -0.0014 & -0.0000 & -0.0000 \\ 0.0094 & 0.0119 & 0.0003 & 0.0016 & -0.0003 & -0.0006 & -0.0086 & 0.0000 & -0.0002 \\ -0.0018 & -0.0023 & -0.0001 & -0.0003 & 0.0001 & 0.0001 & 0.0017 & 0.0000 & 0.0000 \\ -0.0036 & -0.0046 & -0.0001 & -0.0006 & 0.0001 & 0.0003 & 0.0033 & -0.0000 & 0.0001 \\ -0.0492 & -0.0623 & -0.0014 & -0.0086 & 0.0017 & 0.0033 & 0.0446 & -0.0001 & 0.0011 \\ 0.0001 & 0.0002 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0000 & -0.0000 \\ -0.0012 & -0.0015 & -0.0000 & -0.0002 & 0.0000 & 0.0001 & 0.0011 & -0.0000 & 0.0000 \end{bmatrix} \tag{A.10}$$

Covariance matrix of the fictitious system-noise process injected into the system for the loop recovery for the continuous time and discrete time plant are given as follows.

$$W_c = 1000 * I_{9\times 9} + B_p * B_p^T \tag{A.11}$$

$$W_d = B_d * B_d^T \tag{A.12}$$

Covariance matrix of the measurement noise process

$$V_c = 1 * I_{3\times 3} \tag{A.13}$$

$$V_D = 10 \times I_{3\times3} \tag{A.14}$$

where $I_{3\times3}$ is an Identity matrix of dimension (3×3).

$$\text{Stability factor} = \begin{cases} .1\,, & \text{for the Kalman filter} \\ .045\,, & \text{for the Feedback Control law} \end{cases} \tag{A.15}$$

$$\text{Stability factor} = \begin{cases} 1.2\,, & \text{for the Kalman filter} \\ 1.00175\,, & \text{for the Feedback Control law} \end{cases} \tag{A.16}$$

Feedback gain matrix for the discrete time model of the plant

$$F_d = \begin{bmatrix} 0.7110 & 0.6094 & 0.0639 & 0.1028 & 0.0099 & 0.0150 & -0.2497 & 0.0000 & -0.0000 \end{bmatrix} \tag{A.17}$$

Kalman filter gain matrix for the discrete time model of the plant

$$H_d = \begin{bmatrix} 0.0293 & 0.2854 & 0.0007 \\ 0.1849 & -0.0945 & -0.0024 \\ -0.0002 & -0.0013 & 0.0044 \\ 0.0280 & -0.1178 & 0.0031 \\ -0.0306 & 0.0676 & 0.0038 \\ 0.0271 & 0.1837 & -0.0010 \\ -0.0038 & 0.1283 & -0.0020 \\ -0.0034 & 0.0093 & 0.0010 \\ 0.0028 & -0.0293 & 0.0002 \end{bmatrix} \tag{A.18}$$

Feedback gain matrix for the continuous time model of the plant

$$F_c = \begin{bmatrix} 0.8470 & 0.7621 & 0.0789 & -0.0107 & 0.0218 & 0.0062 & -0.3095 & -0.0000 & -0.0001 \end{bmatrix} \tag{A.19}$$

Kalman filter gain matrix for the continuous time model of the plant

$$H_c = \begin{bmatrix} 19.1221 & 32.1700 & 13.4732 \\ 20.8505 & -15.7568 & -20.5089 \\ -2.4111 & 2.5769 & -9.2630 \\ 0.6645 & 0.7940 & 44.4385 \\ -2.0895 & -40.9487 & -0.5780 \\ -5.3863 & 6.0233 & 15.4034 \\ -14.5430 & 14.0600 & -14.6495 \\ -1.3626 & -0.8599 & 15.2589 \\ 0.6439 & -5.7479 & -5.8580 \end{bmatrix} \tag{A.20}$$

LQG_{BD} controller is characterised by the following equations

$$A_c = \begin{bmatrix} 0.1674 & 0.3846 & 0.1731 & 0.1401 & 0.0807 & -0.0003 & -0.0001 & -0.0001 & -0.0000 \\ -0.1187 & 0.0299 & 0.8015 & -0.1410 & -0.1618 & 0.0023 & 0.0001 & 0.0004 & 0.0000 \\ 0.1660 & -0.7486 & 0.3840 & 0.2334 & 0.1703 & -0.0038 & -0.0002 & -0.0007 & -0.0000 \\ 0.0315 & -0.0899 & -0.1824 & 0.7746 & -0.3657 & -0.0036 & 0.0000 & -0.0003 & -0.0001 \\ -0.1724 & -0.0910 & -0.0192 & 0.1053 & 0.0323 & 0.1913 & 0.0060 & 0.0284 & 0.0026 \\ -0.0014 & -0.0016 & -0.0009 & -0.0004 & 0.0048 & -0.2364 & -0.5586 & 0.1879 & 0.0141 \\ -0.0006 & 0.0015 & 0.0007 & 0.0071 & 0.0401 & -0.5062 & 0.6155 & 0.3320 & 0.0216 \\ 0.0004 & -0.0009 & -0.0025 & 0.0025 & 0.0615 & 0.1336 & -0.0285 & 0.4550 & -0.1340 \\ 0.0000 & 0.0004 & 0.0005 & 0.0005 & -0.0039 & 0.0270 & 0.0413 & 0.0644 & 0.8083 \end{bmatrix} \tag{A.21}$$

$$B_b = \begin{bmatrix} 0.3210 & 0.2659 & -0.0000 \\ 0.0862 & -0.1167 & 0.0026 \\ -0.288 & 0.0608 & -0.0008 \\ 0.0449 & -0.0521 & 0.0010 \\ 0.0025 & 0.0244 & 0.0025 \\ 0.0004 & -0.0002 & -0.0157 \\ -0.0000 & -0.0002 & -0.0008 \\ 0.0008 & -0.0010 & -0.0001 \\ -0.0001 & 0.0001 & -0.0003 \end{bmatrix} \tag{A.22}$$

$$C_b = \begin{bmatrix} 0.4262 & 0.0136 & -0.0075 & -0.0180 & -0.0112 & 0.0007 & 0.0000 & 0.0001 & 0.0000 \end{bmatrix} \tag{A.23}$$

$$\Sigma = diag \begin{bmatrix} 0.1894 & 0.0566 & 0.0501 & 0.0199 & 0.0070 & 0.0003 & 0.0002 & 0.0000 & 0.0000 \end{bmatrix} \tag{A.24}$$

$$
T = \begin{bmatrix}
0.4564 & -0.2235 & 0.0376 & -2.0136 & 0.8830 & -0.5198 & 0.8408 & -7.0672 & 19.6624 \\
0.2264 & 0.1181 & -0.0379 & 1.8961 & -0.7744 & 0.6025 & 1.3105 & 17.8265 & -26.3445 \\
-0.1139 & 0.0385 & -0.7463 & -0.1171 & 3.4017 & 0.3046 & 2.4187 & 8.7742 & -8.0685 \\
-0.0772 & 0.6705 & -0.1019 & 0.6624 & -1.0537 & 0.6646 & -12.6232 & -44.3284 & -0.4302 \\
-0.0743 & 0.0223 & 0.8356 & 0.1774 & 2.0831 & 0.1301 & 0.3334 & 1.8231 & -4.3888 \\
0.2693 & -0.8618 & -0.2934 & 0.4389 & 1.2341 & 0.5701 & -5.6150 & -20.8843 & -0.8836 \\
0.1257 & -0.3005 & -0.1357 & -1.0109 & 1.6433 & 0.1392 & 5.2415 & 22.1785 & -10.6217 \\
-0.0908 & 0.1417 & -0.5126 & -0.3818 & 2.9807 & 0.1782 & 3.6211 & 10.4689 & 0.5764 \\
-0.0748 & 0.2645 & -0.1268 & 0.0963 & 1.0268 & 0.4071 & -2.3098 & -8.7941 & -2.0090
\end{bmatrix} \tag{A.25}
$$

LQG_{BS} controller is characterised by the following equations

$$A_b = \begin{bmatrix} -0.2164 & -0.4147 & -0.0947 & 0.0308 & -0.0164 & -0.0013 & 0.0019 & -0.0001 & 0.0000 \\ 0.2764 & 0.2968 & -0.6454 & 0.0206 & 0.0115 & -0.0012 & 0.0012 & -0.0001 & 0.0000 \\ 0.1311 & 0.3832 & 0.1281 & 0.2761 & -0.1850 & -0.0184 & 0.0229 & -0.0015 & 0.0000 \\ 0.0099 & 0.0128 & 0.2043 & 0.6563 & 0.6132 & 0.0130 & -0.0141 & 0.0004 & -0.0000 \\ 0.0643 & -0.1294 & 0.0678 & -0.3304 & 0.3727 & -0.1591 & 0.1273 & -0.0091 & 0.0000 \\ -0.0028 & 0.0159 & 0.0148 & -0.0940 & 0.0879 & 0.9324 & 0.1190 & -0.0084 & 0.0001 \\ -0.0026 & -0.0024 & -0.0371 & 0.0271 & 0.0909 & 0.0045 & 0.2139 & 0.2998 & -0.0025 \\ -0.0014 & 0.0055 & 0.0037 & -0.0217 & 0.0194 & -0.0366 & 0.0783 & 0.2684 & 0.0281 \\ -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0001 & 0.0003 & 0.0000 \end{bmatrix} \tag{A.26}$$

$$B_b = \begin{bmatrix} 0.5910 & 0.1105 & -0.0002 \\ 0.0836 & -0.1589 & -0.0003 \\ 0.0169 & 0.1404 & -0.0002 \\ 0.0072 & -0.0205 & 0.0007 \\ -0.0002 & 0.0116 & 0.0000 \\ 0.0010 & -0.0014 & -0.0004 \\ -0.0014 & 0.0033 & -0.0001 \\ 0.0003 & -0.0005 & -0.0001 \\ 0.0000 & -0.0000 & 0.0001 \end{bmatrix} \tag{A.27}$$

$$C_b = \Big[\; 0.6079 \quad -0.0735 \quad 0.0111 \quad 0.0012 \quad -0.0014 \quad 0.0001 \quad -0.0000$$

$$0.0000 \quad -0.0000 \;\Big] \tag{A.28}$$

$$\Sigma = diag\Big[\; 0.3954 \quad 0.0872 \quad 0.0410 \quad 0.0072 \quad 0.0049 \quad 0.0011 \quad 0.0001$$

$$0.0000 \quad 0.0000 \;\Big] \tag{A.29}$$

$$T = \left[\begin{array}{rrrrrrr} 0.3155 & -0.7322 & 0.1483 & -0.4021 & 1.3179 & -2.0776 & 18.4877 \\ 0.3121 & 0.4132 & -0.2685 & 0.0447 & -1.5454 & 2.7401 & -15.2599 \\ -0.6153 & -0.8412 & -0.7257 & -0.4042 & -4.2401 & -0.9704 & -1.4024 \\ 0.0430 & 0.8230 & 1.0707 & 2.8173 & 1.9664 & 1.7361 & 1.9092 \\ -0.4728 & -0.6562 & -0.9735 & 1.7022 & 1.1095 & 1.3015 & -22.3918 \\ -0.1462 & -0.1575 & 2.4591 & -0.4199 & 0.7119 & 0.4974 & -7.5447 \\ -0.5267 & -1.0415 & -0.5326 & -1.0835 & -1.2514 & 0.8551 & 10.8687 \\ -0.5362 & -0.5047 & -0.9517 & -0.9536 & -2.4443 & -0.3618 & 1.9473 \\ -0.4343 & -0.0596 & 0.1767 & 1.6871 & -0.5358 & 0.7991 & -3.2847 \end{array}\right.$$

$$\left.\begin{array}{rr} -16.2940 & 3.0780 \\ 10.0292 & -9.5175 \\ -12.1829 & -8.8567 \\ 9.4973 & 34.6733 \\ 28.0207 & 9.8945 \\ 10.2746 & 15.4748 \\ -20.7128 & -17.4626 \\ 3.3894 & 11.6382 \\ 11.1771 & -1.3922 \end{array}\right] \tag{A.30}$$

LQG_{FBS} controller is characterised by the following equations

$$A_b = \begin{bmatrix} 0.0192 & -0.8255 & 0.4950 & -0.0818 & -0.2195 & 0.0453 & 0.0274 & -0.0050 & 0.0000 \\ 0.0047 & 0.0259 & 0.9757 & -0.1678 & -0.2874 & 0.0589 & 0.0346 & -0.0063 & 0.0000 \\ 0.0120 & -0.2537 & 0.5449 & 0.5941 & -0.0908 & 0.0258 & 0.0089 & -0.0017 & 0.0000 \\ -0.0753 & 0.1600 & -0.1535 & 0.1298 & -0.6053 & 0.0684 & 0.0620 & -0.0102 & 0.0001 \\ 0.0608 & 0.1122 & 0.0837 & 0.1073 & 0.4951 & -0.1092 & 0.1095 & -0.0157 & 0.0001 \\ -0.0325 & 0.0107 & -0.0043 & -0.0648 & 0.2388 & 1.0117 & 0.0712 & -0.0073 & -0.0000 \\ -0.0021 & -0.0029 & 0.0001 & -0.0418 & 0.0591 & -0.1643 & 0.3609 & 0.3179 & -0.0019 \\ 0.0007 & -0.0044 & -0.0118 & 0.0105 & 0.0276 & 0.0494 & -0.0260 & 0.0567 & 0.1144 \\ -0.0003 & 0.0003 & 0.0016 & 0.0032 & -0.0013 & -0.0217 & 0.0295 & 0.0313 & 0.0081 \end{bmatrix} \tag{A.31}$$

$$B_b = \begin{bmatrix} 17.1194 & 7.6360 & -0.0045 \\ 9.3799 & -3.2504 & -0.0082 \\ -2.6572 & 0.0960 & -0.0029 \\ 1.5383 & 0.6590 & -0.0116 \\ 0.0963 & 0.3932 & 0.0002 \\ -0.0422 & -0.0725 & -0.0026 \\ -0.1116 & 0.0472 & -0.0001 \\ 0.0248 & -0.0687 & -0.0003 \\ -0.0017 & 0.0055 & 0.0001 \end{bmatrix} \tag{A.32}$$

$$C_b = \Big[\ 0.0141 \quad 0.0089 \quad -0.0098 \quad 0.0012 \quad 0.0041 \quad -0.0009 \quad -0.0005$$

$$0.0001 \quad -0.0000\ \Big] \tag{A.33}$$

$$\Sigma = diag\Big[\ 738.7644 \quad 249.0767 \quad 91.1830 \quad 40.035 \quad 10.5598 \quad 2.4909 \quad 0.2072$$

$$0.0240 \quad 0.0003\ \Big] \tag{A.34}$$

$$T = \left[\begin{array}{rrrrrrr}
0.0148 & -0.0021 & 0.0133 & -0.0227 & 0.0388 & -0.0183 & 0.2511 \\
0.0014 & 0.0126 & -0.0043 & 0.0268 & 0.0045 & 0.1290 & -0.0878 \\
-0.0067 & -0.0432 & -0.769 & -0.0827 & -0.3081 & -0.0880 & -0.2634 \\
0.0004 & 0.0068 & -0.0900 & -0.1446 & 0.1751 & 0.3003 & -0.6373 \\
-0.0011 & -0.0110 & 0.0565 & -0.0882 & -0.0319 & -0.1437 & -0.4525 \\
0.0153 & -0.0553 & -0.0101 & 0.0727 & 70.0727 & 0.0152 & -0.5387 \\
-0.0032 & -0.0166 & 0.0449 & -0.0209 & 0.0185 & 0.2278 & 0.3847 \\
-0.0147 & -0.0053 & 0.0221 & -0.0079 & -0.1038 & 0.0918 & -0.1145 \\
-0.0077 & -0.0088 & 0.0064 & 0.0031 & 0.0267 & 0.0196 & 0.3431
\end{array}\right.$$

$$\left.\begin{array}{rr}
-1.8281 & 1.4531 \\
1.4690 & -3.8822 \\
-0.6476 & -3.3247 \\
-0.2966 & 13.4130 \\
2.4475 & 3.4566 \\
0.5548 & 5.8848 \\
-1.3576 & -6.5323 \\
-0.1389 & 4.5064 \\
0.9667 & -0.6756
\end{array}\right] \tag{A.35}$$

LQG_{FBD} controller is characterised by the following equations

$$A_b = \begin{bmatrix} 0.3297 & 0.5776 & 0.4312 & 0.1129 & -0.1462 & 0.2267 & -0.0246 & 0.0026 & -0.0001 \\ -0.1298 & 0.4007 & -0.9380 & -0.0851 & 0.1129 & -0.2466 & 0.0324 & -0.0033 & 0.0001 \\ 0.1549 & 0.4766 & 0.3417 & -0.6675 & 0.0594 & -0.0357 & -0.0157 & -0.0003 & 0.0000 \\ -0.0172 & 0.5163 & -0.1568 & 0.3885 & 0.4690 & -0.4661 & 0.0168 & -0.0044 & 0.0001 \\ 0.0414 & -0.0016 & -0.1692 & -0.1201 & 0.6802 & 0.4219 & 0.1229 & 0.0080 & -0.0006 \\ -0.0678 & 0.2488 & -0.0754 & -0.0545 & -0.1865 & -0.6840 & 0.5309 & 0.0065 & -0.0007 \\ -0.0499 & 0.0252 & 0.0767 & 0.0160 & 0.1643 & 0.0447 & 0.2009 & -0.1380 & 0.0125 \\ -0.0004 & -0.0010 & 0.0002 & -0.0077 & 0.0148 & -0.0477 & -0.0972 & 0.4852 & 0.1785 \\ -0.0002 & 0.0103 & -0.0005 & 0.0062 & 0.0065 & -0.0430 & 0.0359 & -0.1461 & 0.8519 \end{bmatrix} \tag{A.36}$$

$$B_b = \begin{bmatrix} 10.6163 & 9.2961 & 0.0078 \\ -1.4024 & 0.5927 & -0.0234 \\ -2.8344 & -2.5336 & 0.0148 \\ 2.8799 & -1.5713 & 0.0327 \\ -0.9204 & 0.1859 & 0.0076 \\ -0.1463 & 0.3805 & 0.0246 \\ 0.2019 & 0.3284 & 0.0004 \\ -0.0139 & -0.0207 & 0.0000 \\ -0.0270 & 0.0232 & 0.0010 \end{bmatrix} \tag{A.37}$$

Appendix B

In this Appendix the closed loop system matrix and the accompanying perturbation matrices are given.

The closed loop system matrix containing plant and the controller is given as follows

$$\bar{A} = \begin{bmatrix} A_d & -B_d F_d \\ H_d C_d & A_d - B_d F_d - H_d C_d \end{bmatrix}. \tag{B.1}$$

The linear time varying parametric uncertainties in the entries of $\bar{A}$ is given by $\Delta(\bar{A})(k)$

$$\Delta \bar{A}(k) = \begin{bmatrix} \Delta A_d & -\Delta B_d F_d \\ H_d \Delta C_d & 0 \end{bmatrix} \tag{B.2}$$

and

$$\Delta \bar{A}^+(k) = \begin{bmatrix} \Delta A_d^+ & -\Delta B_d^+ F_d \\ H_d \Delta C_d^+ & 0 \end{bmatrix}. \tag{B.3}$$

References

[1] P. P. Khargonekar. K. Poola and A. Tannenbaum."Robust control of linear time invariant plants using periodic compensation." *IEEE Trans Auto. Contr.*, vol.AC-30. no.11. pp.1088-1096. 1985.

[2] D. G. Meyer."A new class of shift varying operators. their shift invariant equivalents. and multirate digital systems." *IEEE Trans. Auto. Contr.*. vol.35. no.4, pp.429-433. 1990.

[3] D. G. Meyer."Cost translation and a lifting approach to the multirate LQG problem." *IEEE Trans Auto. Contr.*. vol.37. no.9. pp.1411-1415. 1992.

[4] Astrom and Wittenmark.*Computer Controlled System: theory and design*, PHI Eaglewood cliffs. N.J. 1984.

[5] E C. Ifeachor and B. W. Jervis.*Digital Signal Processing: A Practical Approach*. Addison-Wesley. wokingham. England

[6] B. D. O. Anderson and Y. Liu,"Controller reduction: Concepts and Approaches." *IEEE Trans. Auto. Contr.*, vol.34. no.8, pp.802-812, 1989.

[7] J. Sklansky and J. R. Ragazzini,"Analysis of errors in sampled data feedback systems-PartII," *AIEE Trans.*, vol.74, pp.65-71, 1955.

[8] G. M. Krank,"Input Output analysis of multirate feedback systems," *IRE Trans. Auto. Contr.*, vol. AC-3, pp.21-28, 1957.

[9] J. R. Ragazzini and G. F. Franklin,"Sampled Data Control System," *New York:McGraw Hill*, 1958.

[10] R. E. Kalman and J. E. Bertram,"A unified approach to the theory of sampling systems." *J. Franklin Inst.*, no.267, pp.405-436. 1959.

[11] J. H. Davis,"Stability conditions derived from spectral theory:Discrete systems with periodic feedback." *SIAM J. Contr.*, vol.AC-29, pp.880-887, 1984.

[12] E. I. Jury and F. J. Mullin,"The analysis of sampled data control systems with periodically time varying sampling rates." *IRE Trans. Auto. Contr.*, vol.AC- 4, pp.15-21, 1959.

[13] R. A. Meyer and C. S. Burrus,"A unified analysis of multirate and periodically time varying digital filters." *IEEE Trans. Circuits Syst.*, vol.CAS-22, pp.162-168, 1975.

[14] B. Sz. Nagy and C. Foias,"Harmonic Analysis of operators on hilbert space." *New York:American Elsevier*, 1970.

[15] P. P. Khargonekar and A. Tannenbaum,"Non euclidian metrics and the robust stabilization of systems with parameter uncertainty." *IEEE Trans. Auto. Contr.*, vol.AC-29, pp.788-793, 1984.

[16] A. Tannenbaum,"Feedback stabilization of plants with uncertainty in the gain factor." *Int. J. Contr.* , vol.32, pp.1-16, 1980.

[17] A. Tannenbaum,"Invariance and system theory: Algebraic and Geometric Aspects." *New York:Springer-verlag*, 1981.

[18] B. Bamieh, J. B. Pearson, B. A. Francis and A. Tannenbaum,"A lifting Technique for linear periodic systems with applications to sampled data control," *Systems and control letters*, vol.17, pp.79-88, 1991.

[19] Y. Yamamoto,"New approach to sampled data control systems: A function space method," *Proc. 29th CDC Honolulu*, pp.1882-1887 Dec.1990.

[20] M. Araki and K. Yamamoto,"Multivariable multirate sampled data systems: State space description. Transfer characteristic, and Nyquist criterion," *IEEE Trans. Auto. Contr.*, vol.AC-31, no.2, pp.145-154 1986.

[21] M. C. Berg, N. Amit and J. D. Powell,"Multirate digital control system design," *IEEE Trans. Auto. Contr.*, vol.33, no.12, pp.1139-1150, 1988.

[22] D. P. Glasson,"A new technique for multirate digital control design and sample rate selection," *AIAA J. Guid. Contr.*, vol.5, pp.379-382, 1982.

[23] H. Al. Rahmani and G. F. Franklin,"A new optimal multirate control of linear periodic and time invariant systems," *IEEE Trans. Auto. Contr.*, vol.35, no.4, pp.406-415, 1990.

[24] H. Al. Rahmani and G. F. Franklin,"Linear periodic systems:Eigenvalue assignment using discrete periodic feedback," *IEEE Trans. Auto. Contr.*, vol.34, pp.99-103, 1989.

[25] A. B. Chammas and C. Leondes,"Pole placement by piecewise constant output feedback," *Int. J. Contr.*, vol.29, pp.31-38, 1979.

[26] P. Colaneri, R. Scattoline and N. Schiavoni,"Stabilization of multirate sampled data linear systems," *Automatica*, vol.26, no.2, pp.377-380, 1990.

[27] B. A. Francis and T. T. Georgiou,"Stability theory for linear time invariant plants with periodic digital controllers," *IEEE Trans. Auto. Contr.*, vol.33, no.9, pp.820-832, 1988.

[28] D. Gangsaas, K. R. Bruce, J. D. Blight and V. L. Ly,"Application of modern synthesis to aircraft control:Three case studies," *IEEE Trans. Auto. Contr.*, vol.AC-31, pp.995-1104, 1986.

[29] D. C. Hyland and D. S. Bernstein,"The optimal projection equations for model reduction and the relationships among the methods of Wilson, Skelton and Moore," *IEEE Trans. Auto. Contr.*, vol.AC-30, pp.1201-1211, 1985.

[30] A. G. Medievski and B. D. O. Anderson."Sampled data Controller reduction procedure."*IEEE Trans. Auto. Contr.*, vol.40, no.11, pp.922-926, 1995.

[31] G. A. Latham and B. D. O. Anderson,"Frequency weighted optimal Hankel-norm approximation of stable transfer functions."*System Contr. Lett.*, vol.5, pp.229, 1985.

[32] S. W. Kim, B. D. O. Anderson and A. G. Medievski."Error bound for transfer function order reduction using frequency weighted balanced truncation."*Syst. Contr. Lett.*, vol.24, pp.183-192, 1995.

[33] B. C. Moore."Principal component analysis in linear systems:Controllability, observability and model reduction."*IEEE Trans. Auto. Contr.*, vol.AC-26, no.1, pp.17-32, 1981.

[34] L. Pernebo and L. M. Silverman."Model reduction via balanced state space representations."*IEEE Trans. Auto. Contr.*, vol.AC-27, no.2, pp.382-387, 1982.

[35] S. Y. Kung and D. W. Lin."Optimal Hankel-norm model reduction:Multivariable systems."*IEEE Trans. Auto. Contr.*, vol.AC-26, no.4, pp.832-852, 1981.

[36] K. Glover."All optimal Hankel-norm approximations of linear multivariable systems and their L^∞-error bounds."*Int. J. Contr.*, vol.39, no.6, pp.1115-1193, 1984.

[37] A. Yousuff, D. A. Wagie and R. E. Skelton."Linear system approximation via covariance equivalent realizations."*J. Math. Anal. Appl.*, vol.106, no.1, pp.91-115, 1985.

[38] A. Yousuff and R. E. Skelton,"Controller reduction by component cost analysis,"*IEEE Trans. Auto. Contr.*, vol.AC-29, no.6, pp.520-530, 1984.

[39] B. D. O. Anderson and R. E. Skelton,"The generation of all q-Markov covers,"*IEEE Trans. Circuits Syst.*, vol.CAS-35, no.4, pp.375-384, 1988.

[40] A. Yousuff and R. E. Skelton,"A note on balanced controller reduction,"*IEEE Trans. Auto. Contr.*, vol.AC-29, no.3, pp.254-257, 1984.

[41] J. A. Davis and R. E. Skelton,"Another balanced controller reduction approximation," *Syst. Contr. Lett.*, vol.4, no.2, pp.79-83, 1984.

[42] B. D. O. Anderson,"Controller design: Moving from theory to practice," *IEEE Contr. Syst. Magazine*, pp.16-25, Aug.1993.

[43] M. G. Safanov, R. Y. Chiang and D. J. N. Limebeer,"Optimal Hankel model reduction for nonminimal systems," *IEEE Trans. Auto. Contr.*, vol.35, no.4, pp.496-502, 1990.

[44] C. P. Therapos,"Balancing transformations for unstable non-minimal linear systems," *IEEE Trans. Auto. Contr.*, vol.34, no.4, pp.455-457, 1989.

[45] J. Keller and B. D. O. Anderson,"A new approach to the discretization of continuous time controllers," *IEEE Trans. Auto. Contr.*, vol.37, no.2, pp.214-223, 1992.

[46] T. Chen and B. A. Francis,"Input output stability of sampled data system," *IEEE Trans. Auto. Contr.*, vol.36, no.1, pp.50-58,1991.

[47] R. K. Yedavalli,"Perturbation bounds for robust stability in linear state space models," *Int. J. Contr.*, vol.42, no.6, pp.1507-1517, 1985.

[48] B. R. Barmish,"Necessary and sufficient condition for quadratic stabilization of an uncertain linear system," *Journal Of Optimization Theory and Application*, vol.46, pp.395-408, 1985.

[49] K. Zhou and P. P. Khargonekar,"Stability robustness bounds for linear state space models with structured uncertainty," *IEEE Trans. Auto. Contr.*, vol.AC-32, pp.621-623, 1987.

[50] K. E. *Holé*,"Design of robust linear quadratic regulator," *American Control Conf., Pitsburgh, USA*, pp.929-930, 1989.

[51] O. I. Kosmidou,"Robust stability and performance of system with structured and bounded uncertainty," *Int. J. Contr.*, vol.52, no.2, pp.627-640, 1990.

[52] J. Cieslik,"On possibilities of the extension of Kharitonov's stability test for interval polynomials to the discrete time case," *IEEE Trans. Auto. Contr.*, vol.AC-32, no.3, pp.237-238, 1987.

[53] A. Vicino,"Some results on robust stability of discrete time systems," *IEEE Trans. Auto. Contr.*, vol.33, no.9, pp.844-847, 1988.

[54] J. M. Martin and G. A. Hewer,"Smallest destabilizing perturbations for linear systems," *Int. J. Contr.*, vol.45, no.5, pp.1495-1504, 1987.

[55] K. M. Sobel, S. S. Banda and H. H. Yeh,"Robust control for linear systems with structured state-space uncertainty," *Int. J. Contr.*, vol.50, no.5,pp.1991-2004, 1989.

[56] Q. L. Han and Q. B. Wu,"Stability robustness analysis of discrete linear systems," *Int. J. Syst. Sci.*, vol.22, no.1, pp.165-172, 1992.

[57] X. Niu, J. A. D. A. Garcia and E. Yaz,"Improved bounds for linear discrete time systems with structured perturbations," *IEEE Trans. Auto. Contr.*, vol.37, no.8, pp.1170-1173, 1992.

[58] J. S. Luo and A. Johnson,"Stability robustness of the discrete time LQG system under plant perturbation and noise uncertainty," *Int. J. Contr.*, vol.55, no.6, pp.1491-1502, 1992.

[59] J. S. Luo, X. F. Zhang and A. Johnson,"Stability robustness in linear state space models with structured time varying uncertainty," *Int. J. Contr.*, vol.60, no.3, pp.435-450, 1994.

[60] B. D. O. Anderson and J. B. Moore, *Optimal Control: Linear Quadratic Methods*, PHI, Englewood Cliffs, New Delhi, 1991.

[61] A. J. Laub, M. T. Heath, C. C. Paige and R. C. Ward,"Computation of system balancing transformations and other applications of simultaneous diagonalization algorithms," *IEEE Trans. Auto. Contr.*, vol.AC-32, no.2 pp.115-121, 1987.

Date Slip

This book is to be returned on the date last stamped.
